Radiative Problems in Black Hole Spacetimes

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Final Defense

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Overview

- Motivation
- RNPL
- Coordinate System
- Massless Scalar Field
- Massive Scalar Field
- 3D Kerr Background
- Future Work

Motivation

Apparent Horizon Boundaries:

- Black holes contain singularities
- Traditional methods don't allow long-time evolutions
- Event Horizon
- Apparent Horizon
- Causal Differencing (Seidel and Suen (1992))

Computer Assisted Programming:

- Homogeneity of finite difference programs
- Difficulty
- Errors
- Must allow complete control over solution methods

RNPL—Rapid Numerical Prototyping Language

RNPL is a language for helping researchers to quickly construct finite-dif ference programs to solve time-dependant systems of partial differential equatio ns.

RNPL can be used to

- Construct the entire program
- Construct everything but the updates

Automatic Features

- Process control, Checkpointing, Interactive I/O
- Parameter management, Memory management
- Parallelization, AMR (Future)
- Ease of Use
- Error free code

Coordinate System

Minimally-Modified Ingoing Eddington-Finkelstein Coordinates (M.W. Choptuik and W.G. Unruh)

$$ds^2 = \left(-\alpha^2 + a^2\beta^2\right) dt^2 + 2a^2\beta dt dr + a^2 dr^2 + r^2 b^2 d\Omega^2$$

$$s = r + f\left(t\right)$$

$$ds^2 = \left(-\alpha^2 + a^2\beta^2\right) dt^2 + 2a^2\beta dt dr + a^2 dr^2 + s^2 d\Omega^2$$

Slicing:

$$\vec{\partial}_t - \vec{\partial}_r \text{ is null}$$

$$g_{tt} - 2g_{tr} + g_{rr} = 0$$

$$\alpha = a\left(1 - \beta\right)$$

Shift:

$$\dot{s} = -\alpha s K^\theta{}_\theta + \beta s'$$

$$\beta = \dot{f} + s\alpha K^\theta{}_\theta$$

$$\beta = \frac{\dot{f} + saK^\theta{}_\theta}{1 + saK^\theta{}_\theta}$$

Equations

Scalar Field:

$$\Pi \equiv \frac{1}{1-\beta}\left(\dot{\phi} - \beta\phi'\right) \qquad \Phi \equiv \phi'$$

$$\dot{\Pi} \equiv \frac{1}{s^2}\left(s^2\left(\beta\Pi + \left(1-\beta\right)\Phi\right)\right)' - 2\,\frac{\dot{s}}{s}\,\Pi - a^2\left(1-\beta\right)\partial_\phi V$$

$$\dot{\Phi} \equiv \left(\beta\Phi + \left(1-\beta\right)\Pi\right)'$$

Geometry:

$$\dot{a} = -a^2\left(1-\beta\right)K^r{}_r + \left(a\beta\right)'$$

$$\dot{K^\theta{}_\theta} = \beta K^\theta{}_\theta{}' + a\left(1-\beta\right)\left(K^\theta{}_\theta\left(K^r{}_r + 2K^\theta{}_\theta\right) - 8\pi V\left(\phi\right)\right) +$$

$$\frac{1-\beta}{s^2}\left(a - \frac{1}{a}\right) + \frac{\beta'}{as}$$

$$K^r{}_r = sK^\theta{}_\theta{}' + K^\theta{}_\theta - \frac{4\pi s\Phi\Pi}{a}$$

Horizon:

$$\left(rb\right)' - arbK^{\theta}{}_{\theta} = 0$$

$$asK^{\theta}{}_{\theta} = 1$$

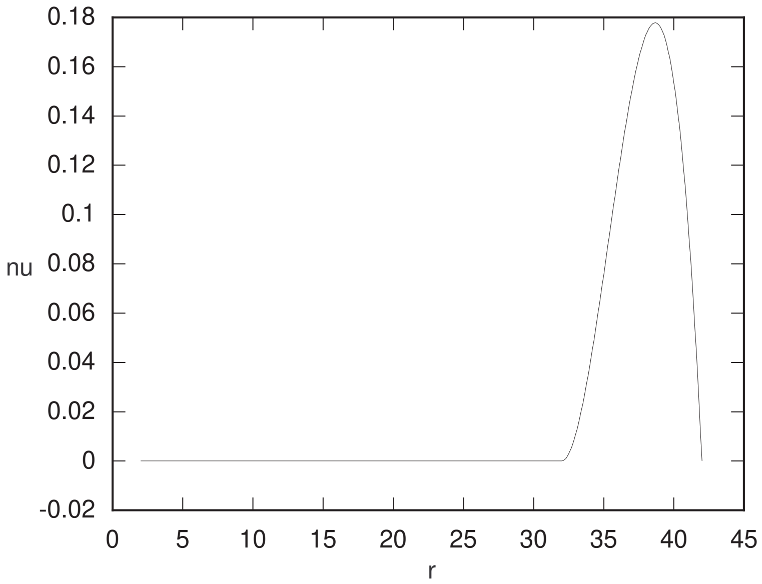
$$\dot{f} = \frac{4\pi s^2 \left(\Phi + \Pi\right)^2}{a^2 \left(1 - 8\pi s^2 V(\phi)\right)}$$

Inner Boundary:

Forward Differencing

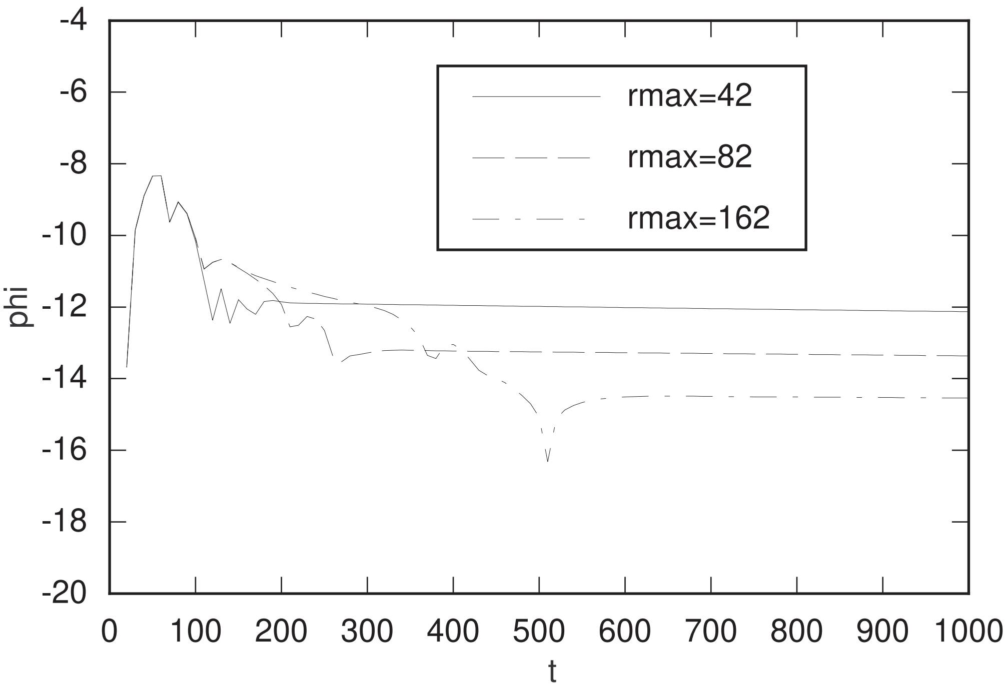
Outer Boundary:

2nd Order Outgoing Radiation Condition with *Sponge Filter*



Power-law Tails

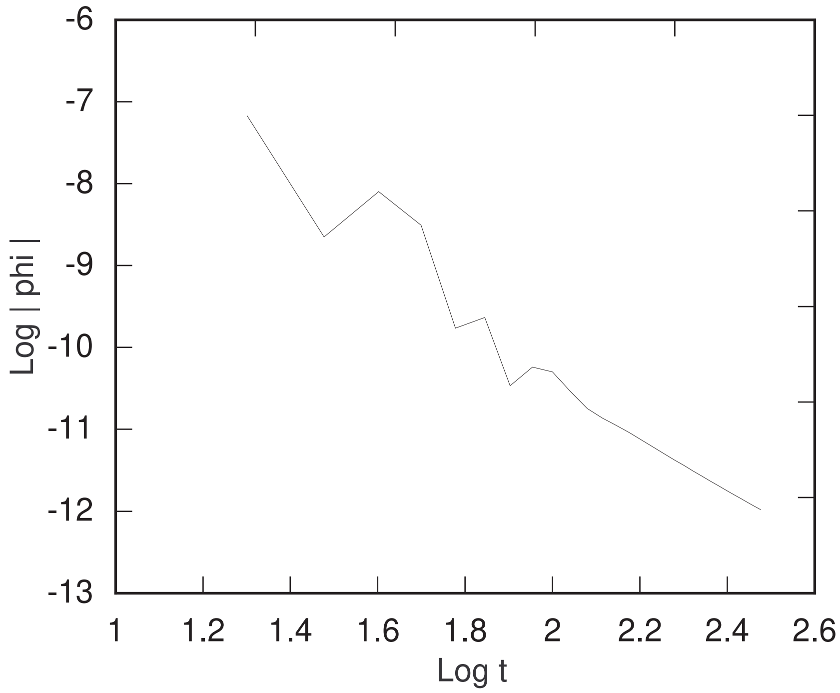
Outgoing radiation is backscattered by the curvature, resulting in a slow fall-off of the scalar field. C. Gundlach et. al. (1994) used perturbation analysis and computer calculations to show that ϕ should go as t^{-3} on the horizon and at large fixed radius.



Log ϕ at $r = 30$ verses t

The power-law fit:

$$\phi(t) = at^{-3.38}$$



Log ϕ at the horizon verses t

The power-law fit:

$$\phi(t) = at^{-3.06}$$

Quasi-normal **Ringing**

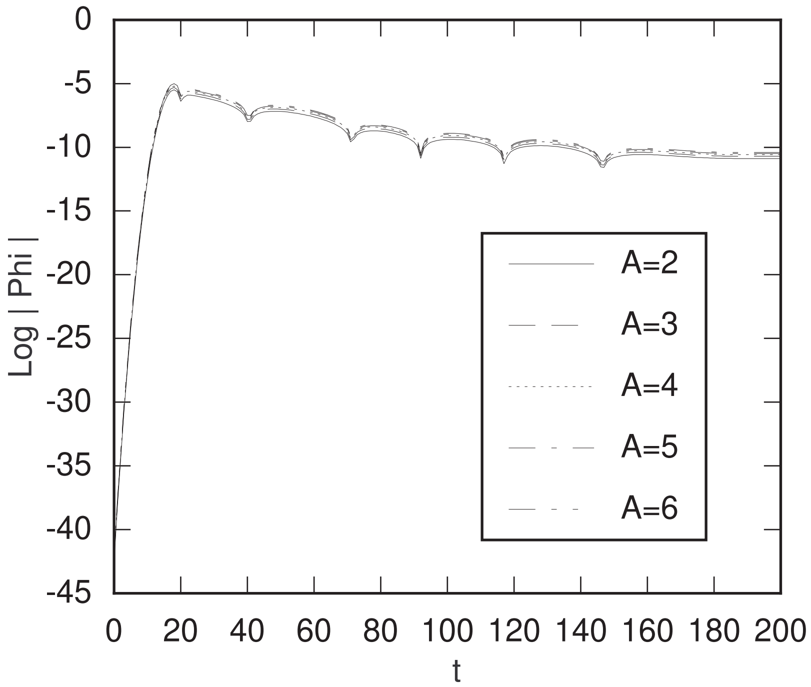
Using perturbation analysis, J. Wheeler (1955) showed perturbations on the Schwarzschild background obey the Regge-Wheeler equation

$$\Phi = \sum_l \frac{1}{r} \Psi_l(t,r)\, Y_{lm}(\theta,\phi)$$

$$\left(-\partial_t^2 + \partial_{r^*}^2\right)\Psi_l = V_{eff}(r)\,\Psi_l$$

$$r^* \equiv r + 2M ln(r - 2M)$$

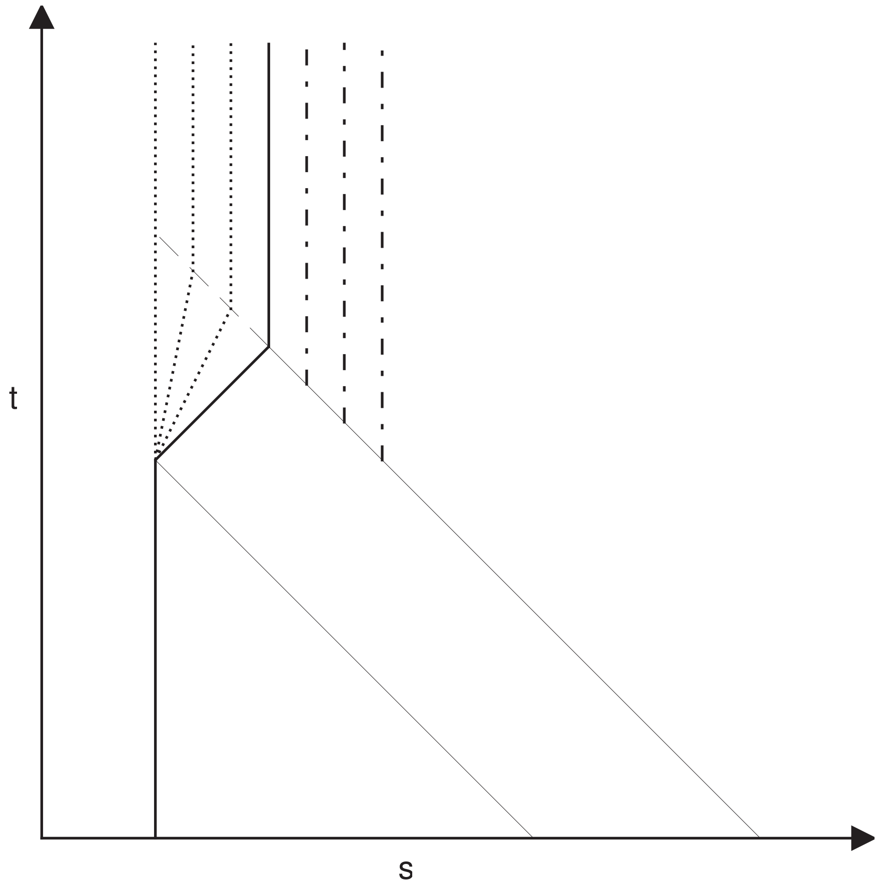
$$V_{eff}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right)$$



Log ϕ at the horizon verses t for various amplitudes

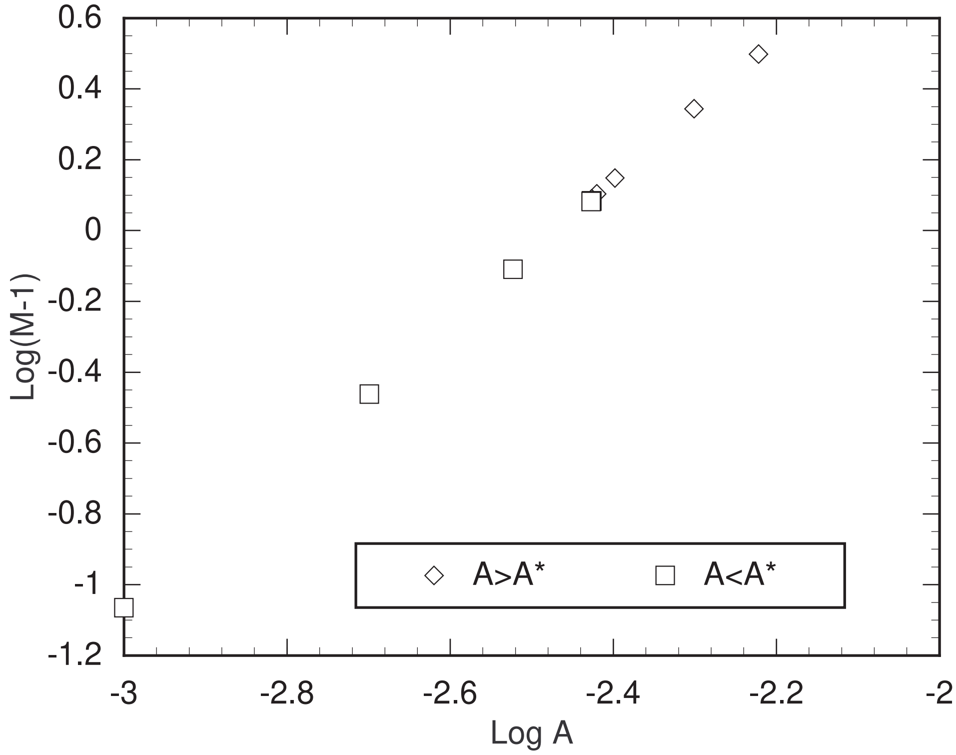
The period for one oscillation is $\approx 53M$ independent of amplitude

Definition of *Criticality*



Schematic motion of the horizon for various amplitudes of scalar field

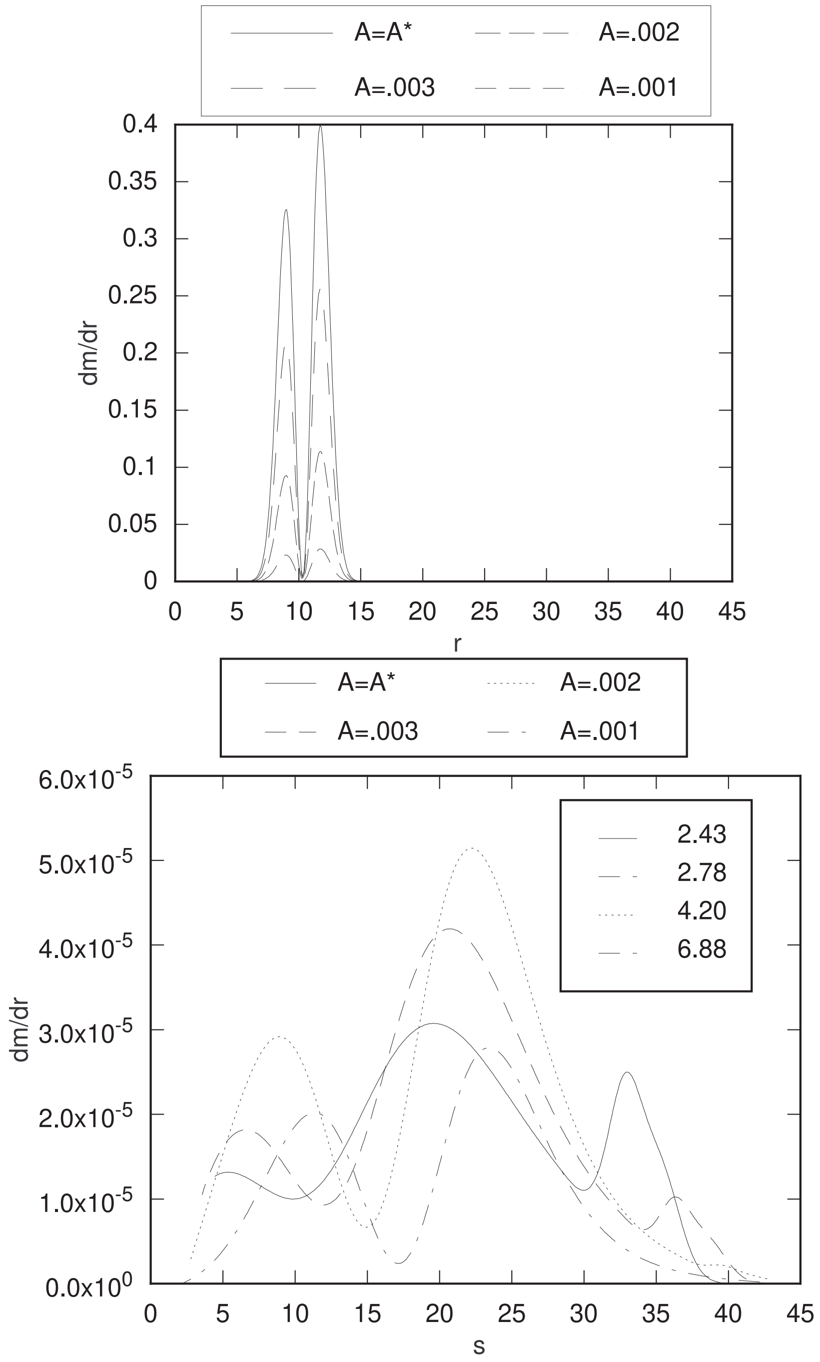
Mass Scaling



Mass verses amplitude

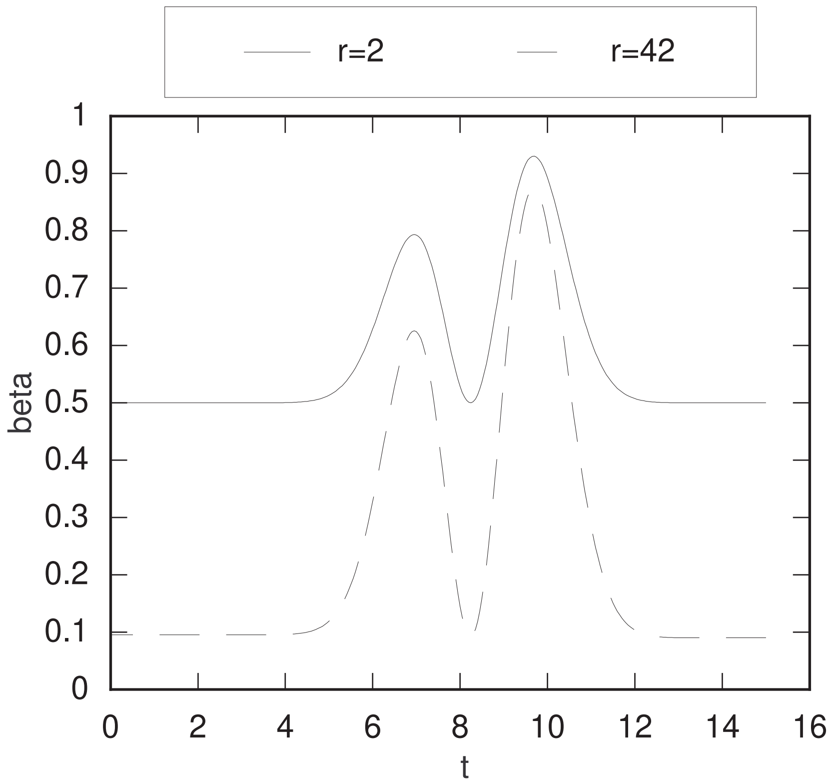
The graph is fit by a line with slope 2.01.

Non-linear Effects



The inset shows 1000 times the fraction of mass scattered.

Coordinate Effects



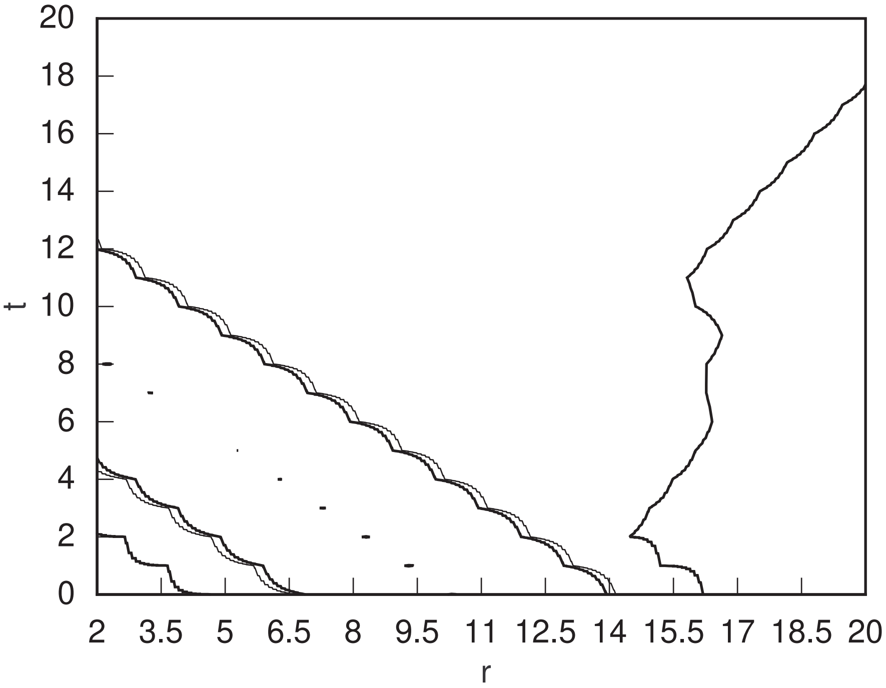
Shift:

$$\beta = \frac{\dot{f} + saK^{\theta}{}_{\theta}}{1 + saK^{\theta}{}_{\theta}}$$

At the horizon:

$$\beta = \frac{\dot{f}}{2} + \frac{1}{2}$$

Characteristic speeds: $-1,\ 1-2\beta$



Evolution of $\frac{dM}{dr}$ showing retrograde motion

Massive Scalar Field

Bechmann and Lechtenfeld (1995) investigated the *Scalar No-Hair Theorem*

Black holes are characterised by mass, charge, and angular momentum. There are no other independent characteristics. A black hole has no "hair" (Wheeler)

The Schwarzschild solution is **the** unique spherically symmetric chargeless solution.

A locally negative potential can yield a static black hole with scalar "hair"

$$ds^2 = -G(\tilde{r})\,d\tilde{t}^2 + G(\tilde{r})^{-1}d\tilde{r}^2 + S(\tilde{r})\,d\tilde{\Omega}^2$$

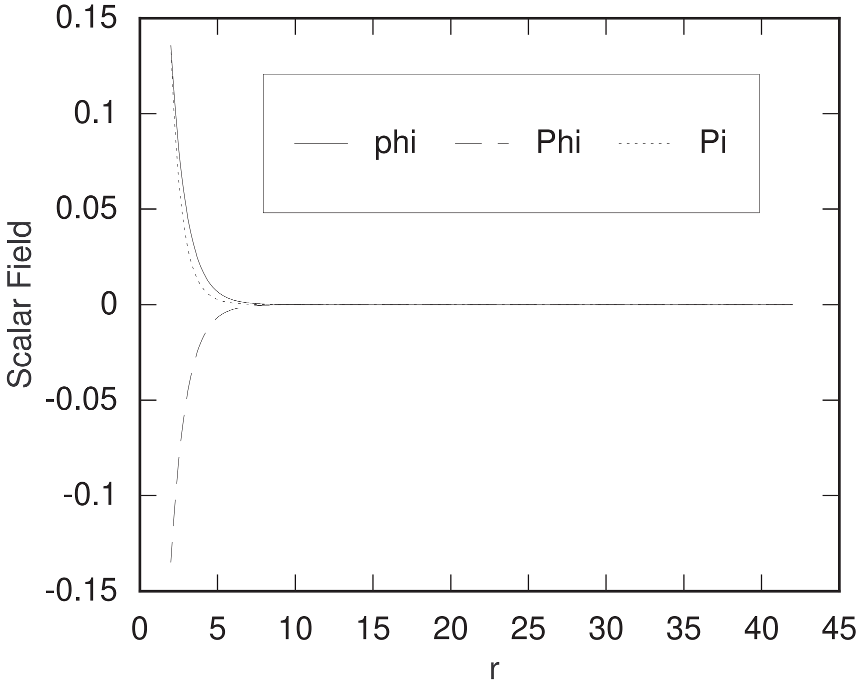
$$\phi(\tilde{r}) = \phi_0\,e^{-m\tilde{r}}$$

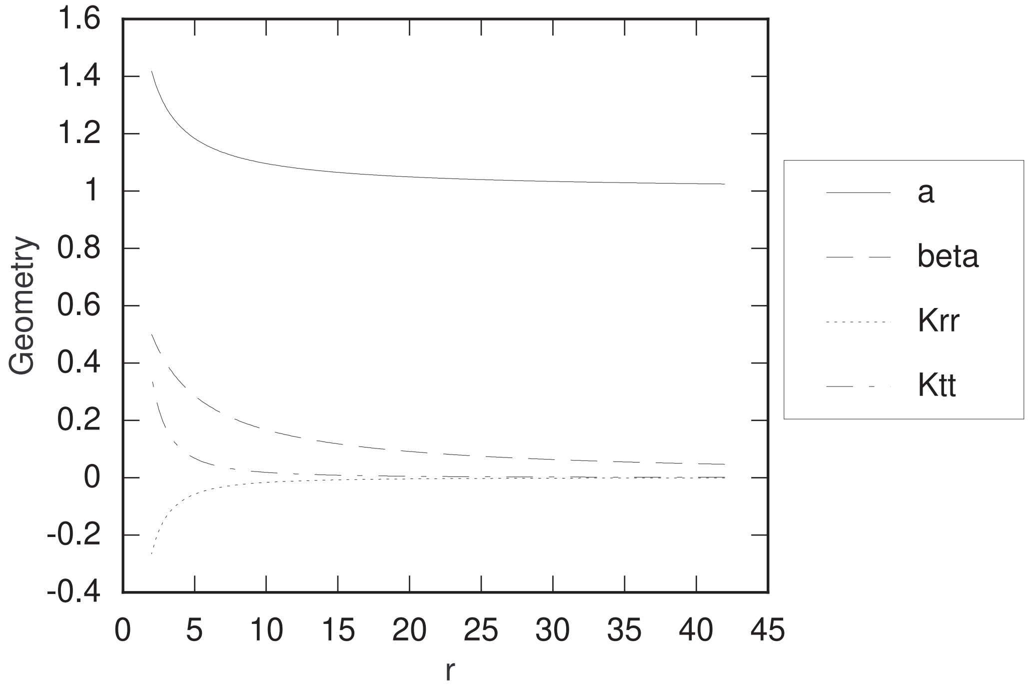
$$S(\tilde{r}) = \frac{1}{m}\left[K_0\left(\frac{1}{2}\,\phi(\tilde{r})\right) + \left(\ln\left(\frac{\phi_0}{4}\right) + \gamma\right)I_0\left(\frac{1}{2}\,\phi(\tilde{r})\right)\right]$$

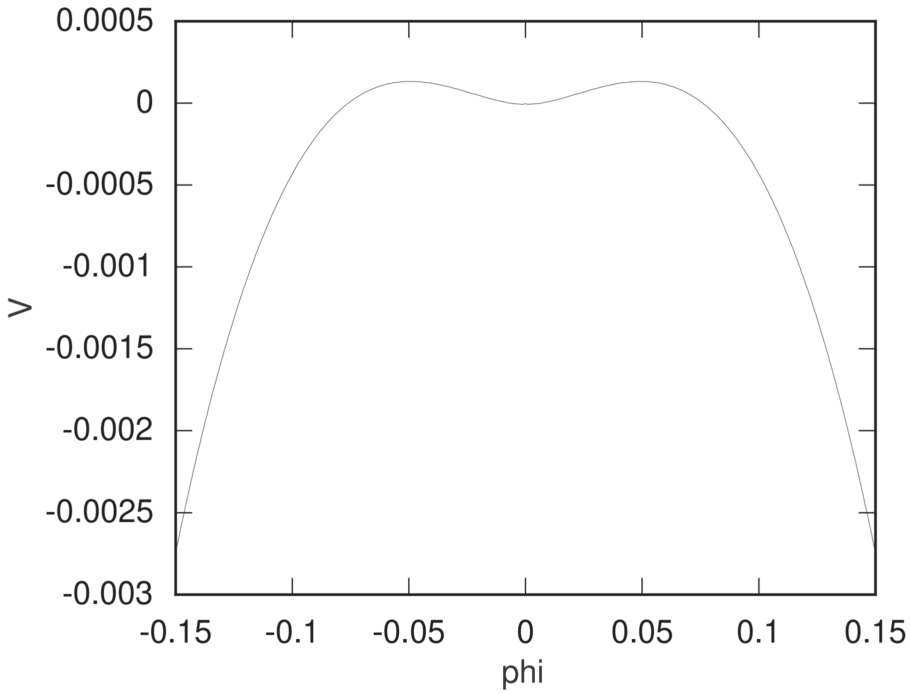
$$G(\tilde{r}) = S^2(\tilde{r})\int_{\tilde{r}}^{\infty}\frac{2\,r' - 6\,M}{S^4(r')}\,dr'$$

$$U(\tilde{r}) = \frac{1}{2}\,G(\tilde{r})\frac{\left(S^4(\tilde{r})\right)''}{S^4(\tilde{r})} + \left(2\,\tilde{r} - 6\,M\right)\left(\frac{1}{S^2(\tilde{r})}\right)' - \frac{2}{S^2(\tilde{r})}$$

$$3M = \frac{\int_h^{\infty} rS^{-4}(r)\,dr}{\int_h^{\infty} S^{-4}(r)\,dr}$$


0.15
0.1
0.05
0
-0.05
-0.1
-0.15
Scalar Field
phi
Phi
Pi
0
5
10
15
20
25
30
35
40
45
r


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
Geometry
a
beta
Krr
Ktt
0
5
10
15
20
25
30
35
40
45
r


0.0005
0
-0.0005
-0.001
-0.0015
-0.002
-0.0025
-0.003
V
-0.15
-0.1
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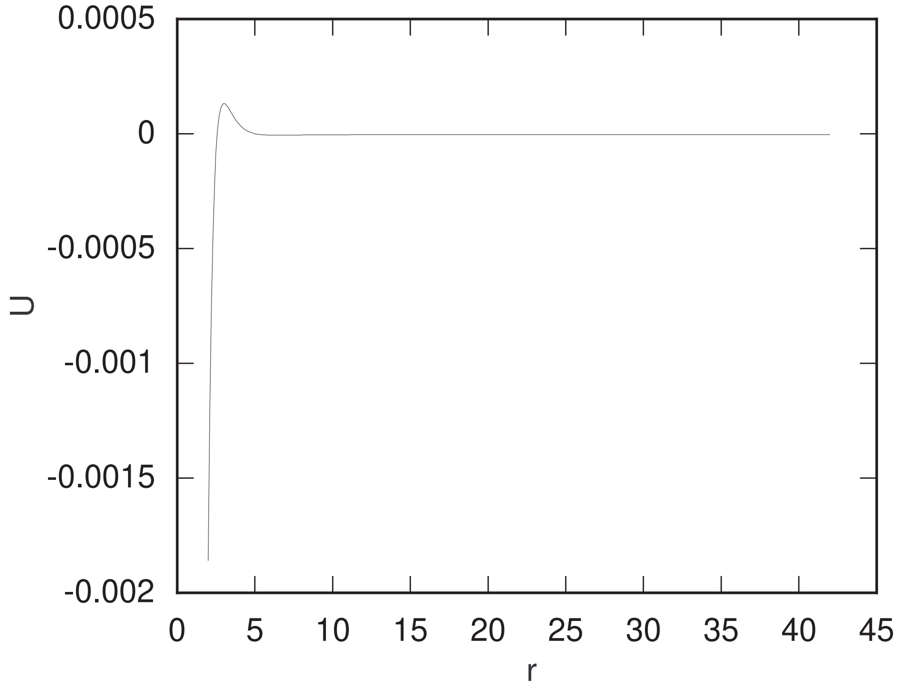
0.15
phi


0.0005
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r

Problems

- Bechmann and Lechtenfeld solution is a *critical* solution
- Can not show convergence directly
- Need a family of solutions with a parameter which interpolates between super- and sub-critical solutions
- May be a difficult problem

The program admits an “arbitrary” potential

Test using a potential of the form $\frac{1}{2}m^2\phi^2$

Kerr Background

Evolution Equations:

$$\Pi \equiv \frac{\sqrt{h}}{\alpha}\left(\partial_t - \beta^i \partial_i\right)\phi$$

$$\Phi_i \equiv \partial_i \phi$$

$$\partial_t \Pi = \partial_i\left(\beta^i \Pi + \alpha\sqrt{h}\left(h^{ij}\Phi_j\right)\right) - \alpha\sqrt{h}\,\partial_\phi V$$

$$\partial_t \Phi_i = \partial_i\left(\frac{\alpha}{\sqrt{h}}\,\Pi + \beta^j \Phi_j\right)$$

Super Radiance:

$$\lambda > \frac{2\,\pi\left(r_+^2 + a^2\right)}{n\,a}$$

$$r_+ = m + \sqrt{m^2 - a^2}$$

Problems

- Lack of Resolution
- Small Spacial Extent

Future Work

- Find an interpolating family of solutions to study the Bechmann and Lechtenfeld solution
- Study the interactions of the massive scalar field with the black hole
- Adaptivize the 3d program and look for superradiance