Neutrino Heating and Baryon Inhomogeneity in Big Bang Nucleosynthesis

by

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To my parents, who have always been there with the love and encouragement that had enabled me to perservere in my work, and to Dr. Matzner, who's always had confidence in my ability.

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In 1993 this author began work on updating and revising the Big Bang Nucleosynthesis code used by Tony Rothman and Richard Matzner of the University of Texas at Austin for a series of articles in the 1980's.

In a temperature range around one GK free neutrons and protons combine to form heavier and heavier nuclei via nuclear reactions and β-decays, with around 25 % of the nucleons forming ^{4}He nuclei. The Texas code originally had a reaction network that could produce abundances $Y(i)$ of nine isotopes, ^{7}Be and lighter. The reaction network was expanded such that the code could then calculate abundances for 68 isotopes. The results were compared with results from the Thomas et al, code, used by David Thomas of the University of Chicago and collaborators for several articles in the 1990's. The codes' calculations of $dY(i)/dt$ and of timesteps were changed so that the mass

fractions of ^{4}He agreed to within 10^{-4}. This author then determined which reactions for each isotope were significant in creating and destroying the isotope, showing which of the heavier isotopes were more significantly produced.

Electrons and positrons transfer energy to neutrinos via annihilation and scattering reactions. This author replaced the Texas code's method of calculating this neutrino heating effect with a method that solves the Boltzmann Transport Equation for the neutrino distribution functions $f_\nu(p)$. The method showed that heating primarily came from e^-e^+ annihilation into $\nu_e\bar{\nu}_e$ pairs and got distributed to μ and τ neutrinos through $\nu_i\nu_j$ reactions, and that $f_\nu(p)$ for higher energy, more tightly coupled neutrinos had the largest percent increases. Increased $f_\nu(p)$ leads to greater conversion of neutrons to protons before nucleosynthesis, but a decrease in electromagnetic plasma temperature T leads to nucleosynthesis occuring earlier and thus with fewer neutrons decayed. The latter effect causes an increase in the mass fraction of ^{4}He by around 1.6×10^{-4} for $\eta_{10} = 3.0$.

The Texas code was for a homogeneous model of the universe. From it this author constructed a code with a spherically symmetric inhomogeneous baryon distribution. Baryon-to-photon ratio η and the model's radius r_i at the initial temperature $T = 100$ GK were varied. Neutrons diffuse from high density regions to low density regions until they are homogeneously distributed. The neutrons then back diffuse when nucleosynthesis depletes neutrons in the high density regions earlier than in the low density regions. How the diffusion rate compared with the neutron-proton conversion rates and the time when nucleosynthesis occured determined the final mass fractions calculated. The inhomogeneous code also included the neutrino heating effect. The change in ^{4}He's mass fraction remained in the range of 1.1-2 $\times 10^{-4}$ for the whole range of η and r_i tested. The effect led to a shift in neutron diffusion to a slightly earlier time that occured in all regions. The heating effect also changed $\log[Y(\mathrm{d})/Y(\mathrm{p})]$ by an order of magnitude of 10^{-3}, an order less than the change it would have by shifting η to raise $X_{^4\mathrm{He}}$. Heating in fact increased deuterium production for the largest values of $r(i)$.

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Chapter 1

Introduction

At the time $t = 0.01$ seconds after the Big Bang nucleonic matter was in the form of free neutrons and protons. Also in existence as free particles were photons, neutrinos and antineutrinos, and electrons and positrons moving at relativistic speeds. Electromagnetic reactions kept electrons/positrons and photons in thermal equilibrium with each other. This electromagnetic plasma had a temperature T which equaled 100 GK at 0.01 s. The neutrino temperature T_ν was 100 GK as well. The neutrons and protons were kept in thermal equilibrium with each other via the following weak reactions, the neutron-proton conversion reactions:

$$\begin{aligned} \mathrm{n} + \nu_{\mathrm{e}} &\leftrightarrow \mathrm{p} + \mathrm{e}^- \\ \mathrm{n} + \mathrm{e}^+ &\leftrightarrow \mathrm{p} + \bar{\nu}_{\mathrm{e}} \\ \mathrm{n} &\leftrightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_{\mathrm{e}} \end{aligned}$$

At $T = 100$ GK these reactions occurred as rapidly in the reverse direction as in the forward, and considerably more rapidly than the expansion rate of the universe. The neutrons and protons were also undergoing reactions that were fusing them into nuclei. The nuclear reactions:

$$
\begin{aligned}
\mathrm{n} + \mathrm{p} &\leftrightarrow \mathrm{d} + \gamma \\
\mathrm{p} + \mathrm{d} &\leftrightarrow {}^{3}\mathrm{He} + \gamma \\
\mathrm{n} + \mathrm{d} &\leftrightarrow \mathrm{t} + \gamma \\
\mathrm{p} + \mathrm{t} &\leftrightarrow {}^{4}\mathrm{He} + \gamma \\
\mathrm{n} +^{3}\mathrm{He} &\leftrightarrow {}^{4}\mathrm{He} + \gamma
\end{aligned}
$$

were the most significant but many other reactions were working as well. These rates were occurring as rapidly in reverse as in forward. So the abundances of heavier nuclei were minimal.

As the time went to 1 s. to 100 s. (T fell from 10 GK to 1 GK) the $\mathrm{e}^{-}\mathrm{e}^{+}$ pairs annihilated each other, mostly forming photons, and T diverged from T_{ν}. The conversion reaction rates fell below the expansion rate, and except for neutron decay they became ineffective . At $t = 100$ s all the nuclear reaction rates fell below the expansion rate. The forward directions of the nuclear reactions depleted the neutrons to form nuclei. The epoch in the Big Bang where the forward directions rapidly occured is known as Big Bang Nucleosynthesis (BBN for short).

In 1993 this author began working on a code that calculated the abundances of nuclei produced by Big Bang Nucleosynthesis. This dissertation refers to that code as the Texas code and this chapter will detail the history of this code soon below. This code had a means of calculating the transfer of energy from electrons and positrons to neutrinos. This neutrino heating effect could slightly alter the Texas code's results. For instance the mass fraction of ^{4}He, $X_{^4\mathrm{He}}$, would increase by an amount in the order of 10^{-4}. Current experimental observations of $X_{^4\mathrm{He}}$ [1] have had errors on the order of 10^{-3}. But in the next few years the accuracy of the measurements might improve to the order of neutrino heating. This author was interested in calculating the heating effect in anticipation of that improved accuracy.

The Texas code needed to be updated though. This code back then calculated the abundances for only nine light isotopes, up to ^{7}Be. Other codes could calculate BBN results for many more isotopes. One such code, hereafter known as the Thomas et al code, will have a detailed description of its background soon below. This author wanted to enable the Texas code to calculate these abundances as well. One needed to see if the calculation of heavy isotope abundances altered the light isotope abundances significantly. Also this author wanted to determine if BBN could produce these heavier isotopes in any interesting amounts. For instance the authors behind the Thomas et al code had some interest in matching the BBN results of ^{9}Be, ^{10}B and ^{11}B to observations. Chapter 2 describes how the author had to change the Texas code and Thomas et al code to bring their abundances to good agreement, and describes the production of heavier isotopes in the Texas code.

Also, improved computation time enabled other authors to develop more sophisticated methods of calculating the neutrino heating effect than the method originally used by the Texas code. This author needed to update the Texas code, and Chapter 3 describes in detail the "MultiBin" method of calculating the effect added to the code, and the results this method yielded.

In the histories behind the Texas code and the Thomas et al code the authors behind both codes developped interests in baryon inhomogeneous models of the universe. The Texas code's researchers wanted to find an inhomogeneous model that satisfied observations and whose overall baryon density equalled the critical density. The Thomas et al code's researchers wanted to see if their inhomogeneous codes produced abundances of ^{9}Be, ^{10}B, ^{11}B and the lighter isotopes that could satisfy observations. No group had looked at the neutrino heating effect in an inhomogeneous code, though. The heating effect's behavior in an inhomogeneous model was not obvious. This author was interested in seeing if the neutrino heating effect changed the abundances of ^{4}He, deuterium or other isotopes in ways that differed from the standard model. The correlation between the changes in ^{4}He and deuterium in the inhomogeneous code turn out to be a distinctive indicator of the heating effect's presence.

1.1 Historical Background

Over the past 50 years researchers have tried to build an accurate numerical model for the BBN process. In the late 1940's Gamow [2] and Alpher & Herman [3] first suggested BBN as a building-up process of successive isotopes starting from a gas of neutrons decaying into protons. Alpher & Herman derived equations for the abundances of isotopes with atomic weight ≤ 4 based on neutron decay and neutron capture processes. But they did not solve the equations because the cosmology for times $t \leq 100$ seconds was poorly understood during the late 1940's. In 1950 Hayashi [4] showed that the two conversion rates other than neutron decay occured more rapidly than the expansion rate at early times, and so significantly determined the amounts of neutrons and protons. Hayashi derived power series equations for the conversion rates and showed that the rates led to nearly equal abundances of neutrons and protons at the highest temperatures. The universe could have enough original protons to make nontrivial amounts of the light elements deuterium, tritium, ^{3}He and ^{4}He.

In 1965 Penzias and Wilson discovered the cosmic microwave background: the remaining free photons from the Big Bang. They measured the photon temperature T_0 initially as 3 K. P.J.E. Peebles used that temperature in his 1966 calculation of the lightest isotopes' mass fractions produced by BBN. In his article [5] Peebles stated the equation for the time evolution of the expansion coefficient R. He argued that the neutrinos expanded adiabatically in the expanding universe. Therefore T_ν should be inversely proportional to R, and R can be assigned an initial value based on T_0. During a run, Peebles would calculate the energy densities and pressures of the various free particles and the neutron proton conversion rates. The rates and the electron energy density he calculated by numerically solving the integral versions of their equations. Peebles also derived equations for the time derivative $dY(i)/dt$ of the abundances $Y(i)$ of the six lightest isotopes (neutrons, protons, deuterium, tritium, ^{3}He, and ^{4}He) based on these five nuclear reactions

$$\begin{aligned}
\mathrm{n} + \mathrm{p} &\leftrightarrow \mathrm{d} + \gamma \\
\mathrm{d} + \mathrm{d} &\leftrightarrow {}^3\mathrm{He} + \mathrm{n} \\
\mathrm{d} + \mathrm{d} &\leftrightarrow \mathrm{t} + \mathrm{p} \\
\mathrm{n} +^3 \mathrm{He} &\leftrightarrow \mathrm{t} + \mathrm{p} \\
\mathrm{d} + \mathrm{t} &\leftrightarrow {}^4\mathrm{He} + \mathrm{n}
\end{aligned}$$

instead of just neutron capture reactions. He integrated these equations numerically as well and calculated the isotope mass fractions $X_i = A_i Y(i)$. (A_i is the atomic weight.) Ultimately, he got final values of $X_{^4\mathrm{He}}$ of 0.256 and 0.282. These values corresponded to the run with a present baryon energy density $\rho_{b0} = 7 \times 10^{-31}\mathrm{g}/cm^3$ (the density in galaxies) and $\rho_{b0} = 1.8 \times 10^{-29}\mathrm{g}/cm^3$ (the accepted value of the critical density in 1966). Both values used 3 K as the present temperature. (These values of ρ_{b0} corresponded to values in the baryon to photon ratio $\eta = [7.7, 198] \times 10^{-10}$, or $\eta_{10} = [7.7, 198]$.)

In a 1967 ApJ article, Wagoner, Fowler and Hoyle (WFH) [6] published the results of a nucleosynthesis code that they could apply to both the universe and to massive stars. Their code had a network of 63 nuclear reaction rates (The code accounted for both the forward and reverse directions of these reactions, as would all later codes for nuclear reactions.), 13 β-decay rates (They calculated only the direction of decay.), and the six neutron-proton conversion rates. The rates applied to 41 different isotopes with ^{26}Al as the heaviest. This network was intended for stellar nucleosynthesis, the process that produces most of the quantities of heavy elements. But this article afforded WFH the opportunity to look at Big Bang heavy element production as well. Their code started at $T = 60$ GK. But at that high a temperature the forward and reverse nuclear rates were much faster than the expansion rate, and nearly equal to each other. This condition was known as Nuclear Statistical Equilibrium (NSE) between the isotopes. The code had to calculate $dY(i)/dt$ and change Y_i to $Y_i + (dY_i/dt)\Delta t$ by timestep Δt. The code could not calculate the very small dY_i/dt or a Δt that would give an accurate value of $Y(i)$. As Peebles [5] did, WFH set $dY_i/dt = 0$ and solved the network for $Y(i)$ at the highest temperatures. From their results WFH concluded that the production of ^{4}He, deuterium, ^{3}He and ^{7}Li came largely from BBN, but not the heavier isotopes. Their results for those light elements agreed with measured results at $\rho_{b0} = 2 \times 10^{-31}\text{g}/cm^3$, (an η_{10} of about 2.2).

In 1969, R.V. Wagoner [7] reported on a nucleosynthesis code with a network of 185 nuclear reaction rates and 29 β-decay rates, and the neutron-proton conversion rates. The code had 61 different isotopes with ^{32}S as the heaviest species. As with the code in WFH, this code could be applied to any scenario that starts as a high temperature. Wagoner wrote the code such that he could plug in scaling factors for the expansion coefficient R, the current baryon density ρ_{b0}, and the initial difference between the neutron and proton mass fractions $X_{\text{p}} - X_{\text{n}}$. He varied those factors to collect results for a variety of scenarios. He would usually set X_{p} and X_{n} nearly equal though, and start at an initial $T = 10$ GK. The BBN codes discussed in this dissertation start at $T = 100$ GK, and by the time they reach 10 GK X_{p} and X_{n} have the values of around 0.75 and 0.25 respectively. The results Wagoner reported had no relevance to this dissertation. In Appendix C of the article, though, Wagoner derived an equation for $dY(i)/dt$ linearized in terms of the $Y(i)$. This linearized expression enabled him to calculate $dY(i)/dt$ at the highest temperatures and for the heaviest isotopes, and he could then use the second-order Runge-Kutta method to get the next timestep's value for $Y(i)$. The appendix also showed how to calculate Δt so that it could stay small enough to calculate $Y(i)$ accurately. Many later BBN codes (SW, 1977 [8]; OSSTY, 1981 [9]; BS, 1985 [10]; KSS, 1988 [11]) would utilize this same method to solve for $Y(i)$. The dissertation describes Wagoner's means of calculating $dY(i)/dt$ and Δt in detail in Chapter 2.

1.2 The Thomas et al Code: History

For a 1991 article, WSSOK [12] used a code with a total number of 31 reaction rates to calculate $Y(i)$ up to $i =^7$ Be. They cited recent improved measurements of some of the reaction rates, the neutron lifetime τ_n, and the number of neutrino species N_ν as their motivation. So they ran their code with the latest expressions for the reaction rates, and solving the integral forms of the neutron-proton conversion rates as τ_n had become known more accurately than the power series expressions. They compared their results with the results by KSS [11], graphing the differences in mass fractions and abundances. (For instance the new rates increased $X_{^4\mathrm{He}}$ by 0.0011 for all values of η_{10}.) WSSOK also wrote extensively about the latest experimental measurements of X_i (or $Y(i)/Y(\mathrm{p})$) of ^{4}He, deuterium, ^{3}He and ^{7}Li, establishing boundaries of 2.8 $\leq \eta_{10} \leq$ 4.0 and $N_\nu \leq$ 3.3.

Two years later, TSOF [13] expanded the code to calculate X_i of isotopes heavier than ^{7}Be, in response to recent measurements of Be and B in stars and limits to the amount of ^{6}Li. Their code, the **Thomas et al code**, had a network of 180 reaction rates applied to 45 different isotopes. TSOF claimed that they had the most comprehensive network for isotopes with $A_i >$ 6 ever applied to the SBBN model. For the range of η_{10} from WSSOK [12] their results for $Y(^9\mathrm{Be})/Y(\mathrm{p})$ and $Y(^{11}\mathrm{B})/Y(\mathrm{p})$ were four orders of magnitude too small to match measured results. But their results could increase four orders of magnitude when they increased the neutron to proton ratio for T in a range of 5 GK down to 3 GK. The production of the lighter isotopes would also increase considerably. But TSOF speculated that they could balance all the results out with an inhomogeneous BBN model pairing zones with n/p this high with zones of very low n/p.

Thomas on his own posted a followup article [14] to the Los Alamos Archives. He graphed the Thomas et al code's results for $Y(^9\mathrm{Be})/Y(\mathrm{p})$, $Y(^{10}\mathrm{B})/Y(\mathrm{p})$, and $Y(^{11}\mathrm{B})/Y(\mathrm{p})$ for $1.0 \geq \eta_{10} \geq 10.0$. The graphs included 2σ error bounds based on the uncertainties of 12 reactions reported in Smith, Kawano, and Malaney, 1993 [15] and calculated with a Monte-Carlo method described in SKM. The abundance $Y(^9\mathrm{Be})/Y(\mathrm{p})$ for this range of η_{10} was still smaller than the experimental observations Thomas quoted, by two orders of magnitude. Thomas's measurements for B were only for the total B in stars. But the code's amount of ^{10}B was much smaller than that of ^{11}B for these η_{10}. Still $Y(^{11}\mathrm{B})/Y(\mathrm{p})$ was too small also by two orders of magnitude.

TSOF along with Mathews and Meyers were finally able to use the Thomas et al code for an inhomogeneous BBN model in a 1994 article [16]. For this article the code had 91 new reaction rates yielding abundances for a total of 68 isotope species. They made the code into an inhomogeneous multizone code based on the work of Matthews, Meyer, Alcock and Fuller [17], with a lattice of spheres consisting of 16 spherical shells in decreasing value of ρ_b. They graphed the resultant abundances dependent on the average η_{10} and on the separation between spheres $2r$. They found that a range of $2.8 \leq \eta_{10} \leq 3.3$ and $r \leq$ one meter yielded abundances of deuterium, ^{3}He, ^{4}He and ^{7}Li that agreed with experimental measurements. Within that range of r the abundances for ^{6}Li, ^{9}Be, ^{10}B, and ^{11}B did not change from their SBBN results. So TSOMMF concluded that the production of these four isotopes was something other than BBN.

1.3 The Texas Code: History

Independently from Thomas et al, Tony Rothman began working on a BBN code, the **Texas code**, in 1981 [18]. His dissertation [18] and three subsequent papers he cowrote with Richard Matzner (RM, 1982a [19]; RM, 1982b [20]; RM, 1984 [21]) calculated the mass fractions of deuterium and ^{4}He in models with an anisotropic distribution of neutrinos. Their anisotropic model used a Bianchi Type-I metric taken from a 1972 article by Matzner and Misner [22] which would lead to a physical anisotropy $\hat{\beta}$ and a neutrino anisotropy potential $V(\hat{\beta})$. In (RM 1982a) [19] they derived equations for the expansion rate $\dot{\alpha} = \dot{R}/R$ and the time derivative of $\rho_{e+\gamma}$ of the electrons/positrons plus the photons (electromagnetic plasma) that included terms dependent on $\hat{\beta}$ and $V(\hat{\beta})$. In their model the anisotropic electron neutrinos interacted with $\mathrm{e}^-\mathrm{e}^+$ pairs by the reaction $\nu_\mathrm{e} + \bar{\nu}_\mathrm{e} \rightarrow \mathrm{e}^- + \mathrm{e}^+$, but only by that reaction, and their equation for $\dot{\rho}_{\mathrm{e}+\gamma}$ had a term dependent on neutrino energy density ρ_ν and $1/t(\nu_\mathrm{e} + \bar{\nu}_\mathrm{e} \rightarrow \mathrm{e}^- + \mathrm{e}^+)$, the collision time of that reaction. Their collision time was proportional to a parameter K that they varied so that the reaction would fall below $\dot{\alpha}$ at different times. Varying K and $\hat{\beta}$ they compiled resulting mass fractions of deuterium and ^{4}He, which were calculated with a network of only six nuclear reactions.

Rothman and Matzner redid their calculations in (RM 1984) [21] using a more sophisticated and more detailed version of the Texas code. Their new version distinguished between the three different neutrino species, with a $V(\hat{\beta}_\mathrm{e})$, $V(\hat{\beta}_\mu)$ and $V(\hat{\beta}_\tau)$, and terms in the equations for $\dot{\alpha}$ and $\dot{\rho}_{\mathrm{e}+\gamma}$ for each of the three neutrino species. Further, along with $\nu\bar{\nu}$ annihilation into $\mathrm{e}^-\mathrm{e}^+$, the code accounted for neutrino scattering off of electrons/positrons, other neutrinos of a different species, and other neutrinos of the same species. For all these collision times the code replaced K with the actual expression for $1/t$ calculated from $n_\nu \sigma c$, the cross section times the neutrino number density times c. In (RM 1982a) [19] Rothman and Matzner assumed that $\rho_\nu R^4$ stayed constant all through a run for all neutrino species. This assumption held if the neutrinos had no interactions with any other particles. In (RM 1984) the code had many such interactions. So Rothman and Matzner derived equations for the time derivative of $\rho_\nu R^4$, with different equations for $d(\rho_{\nu_e} R^4)/dt$, $d(\rho_{\nu_\mu} R^4)/dt$, and $d(\rho_{\nu_\tau} R^4)/dt$. The time derivative equations for α and the ρ's were the equations Rothman and Matzner evolved using second-order Runge-Kutta. They noted in their article that this method of evolving the code differed from the method used in (Wagoner 1969) [7], resulting in mass fractions of ^{4}He and deuterium that differed by 0.4 % and 4.0 % respectively. Finally, they increased the number of nuclear reactions to 15. They felt that they needed only these rates to calculate accurate mass fractions for deuterium and ^{4}He. Ultimately, they found that even with small anisotropy potentials these two mass fractions increased beyond the ranges of experimental measurements taken at that time. In a 1985 follow-up article, Kurkio-Suonio and Matzner [23] extended the network to 30 rates to calculate the results for ^{7}Li as well. Kurki-Suonio and Matzner found linear relationships between the anisotropy and the abundances for small anisotropy.

Matzner reviewed the results of modifications to the standard BBN model in a 1986 article [24]. Along with restating the anisotropic results from (KM, 1985) [23], Matzner mentioned results for a model he worked on with Joan Centrella et al (CMRW, 1986) [25] that had plane-symmetric inhomogeneous distributions for all its particles. A 1988 article (KMCRW, 1988) [26] began a series of articles where Kurki-Suonio, Matzner, et al tried to find an inhomogeneous model that would have $\rho_b =$ the critical density for a closed universe and give abundances of deuterium and ^{4}He that agreed with observations. In the standard code $X_{^4\mathrm{He}}$ at critical density was on the order of 0.04 larger than observed values. The articles' authors hoped to reduce that value in an inhomogeneous model.

In (KMCRW, 1988) the authors combined the inhomogeneous code from (CMRW, 1986) with the Texas code to make a code that was one-dimensional plane-symmetric inhomogeneous only in its baryon density. The code started out with five zones with a high density of and 15 zones with a low density. During a run, the code could calculate the diffusion of neutrons between the zones. Their results showed that at first neutrons diffused from the high density zones into the low density zones. But the neutrons would then diffuse back as nucleosynthesis consumed the neutrons in the high density zones to ultimately form ^{4}He. So the neutrons would wind up concentrated along the boundaries between the high and low density regions, and $X_{^4\mathrm{He}}$ averaged out over the zones did not differ much from its standard value.

For a 1989 article, Kurki-Suonio and Matzner [27] replaced the previous article's code with a code streamlined to run much faster. This new code, still modelled on the Texas code, could calculate results for spherical and cylindrical geometries as well as planar, and had a minimum of 64 zones. The authors ran this code for density contrasts R_ρ between high and low density regions of 100 or less and varied the distance scale r_i of the model. As they increased r_i, $X_{^4He}$ would decrease, but increase again once r_i got large enough to minimize neutron diffusion. But still the decrease was only on the order of 0.01, too small to reach observational values.

Kurki-Suonio et al used the new inhomogeneous code for a 1990 Astrophysical Journal article (KMOS, 1990) [28]. Instead of calculating the abundances for a critical density universe, they plotted on an r_i vs η_{10} contour map the range of r_i and η_{10} that would give abundances that agreed with observations. For η_{10} between 2.2 and 7 they were able to find an r_i that satisfied observations. Then in a 1990 Physical Review D article Kurki-Suonio and Matzner [29] went over the neutron diffusion equation in detail. In this paper they modified the model to be able to have a nonuniform grid of 64 zones, explaining how to define this grid. The nonuniform grid enabled them to do runs for R_ρ up to 10240. Their results showed that models with only 8 or 16 zones yielded inaccurate abundances for r_i around 10 light-hours. For R_ρ higher than 100 they needed more resolution than 64 uniform zones could provide, but the nonuniform grid did provide that resolution. $X_{^4\mathrm{He}}$ still did not decrease enough from its standard model value to reach observations, in fact decreasing less than in the coarser grids.