# In the Texas Code:

First Runge-Kutta Step

$$
\begin{aligned}
Y_n(i) &= Q_{ij}(Y_n, [ij](T_n), [kl](T_n)) \times Y_{n\alpha}(j) \\
\frac{dY(i)}{dt}(t, Y_n) &= \frac{Y_{n\alpha}(i) - Y_n(i)}{\Delta t_n} \\
Y_{n\alpha}(i) &\approx Y_{n+1}(i)? \\
\tilde{Y}_n(i) &= Y_n(i) + \frac{Y_{n\alpha}(i) - Y_n(i)}{\Delta t_n} \Delta t_n \\
\tilde{Y}_n(i) &= Y_{n\alpha}(i)
\end{aligned}
$$

Second Runge-Kutta Step

$$
\begin{aligned}
Y_n(i) &= Q_{ij}(\tilde{Y}_n, [ij](T_{n+1}), [kl](T_{n+1})) \times Y_{n\beta}(j) \\
\frac{dY(i)}{dt}(t + \Delta t_n, \tilde{Y}_n) &= \frac{Y_{n\beta}(i) - Y_n(i)}{\Delta t_n} \\
Y_{n\beta}(i) &\approx Y_{n+1}(i)? \\
Y_{n+1}(i) &= Y_n(i) + \frac{1}{2}\left[\frac{Y_{n\alpha}(i) - Y_n(i)}{\Delta t_n} + \frac{Y_{n\beta}(i) - Y_n(i)}{\Delta t_n}\right] \Delta t_n \\
Y_{n+1}(i) &= \frac{1}{2}(Y_{n\alpha}(i) + Y_{n\beta}(i))
\end{aligned}
$$

The schemes imply that both $Y_{n\alpha}(i)$ and $Y_{n\beta}(i)$ are approximations of $Y_{n+1}(i)$, and that the first time derivative corresponds to $dY(i)/dt(t + \Delta t_n, Y_n(i))$ instead of $dY(i)/dt(t, Y_n(i))$

Further, the actual value of $Y_{n+1}(i)$ winds up the average of $Y_{n\alpha}(i)$ and $Y_{n\beta}(i)$, both of which are first-order approximations.