To avoid the Boltzmann equation, assume that the $f_\nu(p)$'s are equiproportionally heated for all p (one bin).

Then at a time t assume $f_\nu(p)$ has energy transferred to it from the $f_e(p)$ at the same p from all time $t' < t$. The amount from t' is determined by a weight function $W_\nu(\alpha';\alpha)$ So:

$$
\begin{aligned}
\rho_\nu &= \frac{15}{\pi^4} a \left(\frac{T_N}{R}\right)^4 \int dp p^3 \int_{-\infty}^{\alpha} d\alpha' \frac{W_\nu(\alpha';\alpha)}{e^{\frac{p}{T}\frac{T_N}{R}}+1} \\
\rho_\nu &= \frac{7}{8} a \left(\frac{T_N}{R}\right)^4 \int_{-\infty}^{\alpha} d\alpha' \left(R'\frac{T}{T_N}\right)^4 W_\nu(\alpha';\alpha) \\
\rho_\nu &= \rho_{\nu_0} \int_{-\infty}^{\alpha} d\alpha' \left(R'\frac{T}{T_N}\right)^4 W_\nu(\alpha';\alpha) \\
\rho_\nu R^4 &= \frac{7}{8} a T_N^4 \int_{-\infty}^{\alpha} d\alpha' \left(R'\frac{T}{T_N}\right)^4 W_\nu(\alpha';\alpha)
\end{aligned}
\tag{1}
$$

using W_{ν_e} and W_{ν_μ} for f_{ν_e} and f_{ν_e}. Similarly for n_ν

$$
\begin{aligned}
n_\nu &= \frac{15}{\pi^4}\frac{a}{k} \left(\frac{T_N}{R}\right)^3 \int dp p^2 f_\nu(p) \\
n_\nu &= \frac{15}{\pi^4}\frac{a}{k} \left(\frac{T_N}{R}\right)^3 (1.803) \int_{-\infty}^{\alpha} d\alpha' \left(R'\frac{T}{T_N}\right)^3 W_\nu(\alpha';\alpha) \\
n_\nu R^3 &= \frac{15}{\pi^4}\frac{a}{k} T_N^3 (1.803) \int_{-\infty}^{\alpha} d\alpha' \left(R'\frac{T}{T_N}\right)^3 W_\nu(\alpha';\alpha)
\end{aligned}
$$

Make the approximation that the $\rho_\nu R^4$ and $n_\nu R^3$ terms decay exponentially by the reactions that deplete them. So one can write an equation for $n_{\nu_i} R^3$ by assuming that $n_{\nu_i} R^3$ is exponentially decayed by reactions that annihilate ν_i

$$\begin{aligned}
\frac{d}{dt}(n_{\nu_e} R^3) \quad &= \quad -\left(\frac{n_{\nu_e} R^3}{t(\nu_e \bar{\nu}_e \to e^- e^+)} + \frac{n_{\nu_e} R^3}{t(\nu_e \bar{\nu}_e \to \nu_\mu \bar{\nu}_\mu)} + \frac{n_{\nu_e} R^3}{t(\nu_e \bar{\nu}_e \to \nu_\tau \bar{\nu}_\tau)}\right) \\
&\quad + \frac{n_{\nu_\mu} R^3}{t(\nu_\mu \bar{\nu}_\mu \to \nu_e \bar{\nu}_e)} + \frac{n_{\nu_\tau} R^3}{t(\nu_\tau \bar{\nu_\tau} \to \nu_e \bar{\nu}_e)} + \frac{n_e R^3}{t(e^- e^+ \to \nu_e \bar{\nu}_e)} \qquad (2)
\end{aligned}$$

Similarly for $\rho_{\nu_i} R^4$

$$\frac{d}{dt}(\rho_{\nu_e} R^4) \quad = \quad -\frac{\rho_{\nu_e} R^4}{t(\nu_e \to)} + \frac{\rho_{\nu_e} R^4}{t(\nu_e \to \nu_e)} + \frac{\rho_{\nu_\mu} R^4}{t(\nu_\mu \to \nu_e)} + \frac{\rho_{\nu_\tau} R^4}{t(\nu_\tau \to \nu_e)} + \frac{\rho_e R^4}{t(e \to \nu_e)} \qquad (3)$$

Similar equations hold for $\rho_{\nu_\mu} R^4$ and $\rho_{\nu_\tau} R^4$, and $n_{\nu_\mu} R^3$ and $n_{\nu_\tau} R^3$. Evolve these equations using 2nd. Order Runge-Kutta.

$t(\nu_e \rightarrow)$ accounts for average reaction times with ν_e as an incoming particle.

$$
\begin{aligned}
\frac{1}{t(\nu_e \rightarrow)} \quad = \quad & \frac{1}{t(\nu_e\nu_e \rightarrow \nu_e\nu_e)} + \frac{1}{t(\nu_e\bar{\nu}_e \rightarrow \nu_e\bar{\nu}_e)} + \frac{1}{t(\nu_e\nu_\mu \rightarrow \nu_e\nu_\mu)} \\
& + \frac{1}{t(\nu_e\bar{\nu}_\mu \rightarrow \nu_e\bar{\nu}_\mu)} + \frac{1}{t(\nu_e\nu_\tau \rightarrow \nu_e\nu_\tau)} + \frac{1}{t(\nu_e\bar{\nu}_\tau \rightarrow \nu_e\bar{\nu}_\tau)} \\
& + \frac{1}{t(\nu_e\bar{\nu}_e \rightarrow \nu_\mu\bar{\nu}_\mu)} + \frac{1}{t(\nu_e\bar{\nu}_e \rightarrow \nu_\tau\bar{\nu}_\tau)} + \frac{1}{t(\nu_e e \rightarrow \nu_e e)} \\
& + \frac{1}{t(\bar{\nu}_e e \rightarrow \bar{\nu}_e e)} + \frac{1}{t(\nu_e\bar{\nu}_e \rightarrow e^- e^+)}
\end{aligned}
$$

$t(\nu_\mu \rightarrow \nu_e)$ for example accounts for average reaction times with ν_μ as an incoming particle and ν_e as an outgoing particle.

$$
\frac{1}{t(\nu\mu \rightarrow \nu_e)} \quad = \quad \frac{1}{2t(\nu_\mu\nu_e \rightarrow \nu_\mu\nu_e)} + \frac{1}{2t(\nu_\mu\bar{\nu}_e \rightarrow \nu_\mu\bar{\nu}_e)} + \frac{1}{2t(\nu_\mu\bar{\nu}_\mu \rightarrow \nu_e\bar{\nu}_e)}
$$

(The factor of 2 indicates that the incoming energy is evenly distributed between the outgoing particles.) Reaction times are determined from average cross sections $1/t = cn_\nu\sigma$.

Define a central temperature $T_{\nu_e c}$ such that

$$
\begin{aligned}
T_{\nu_e c} &= \frac{\rho_{\nu_e}}{n_{\nu_e}} \frac{\frac{15}{\pi^4}\frac{a}{k}(1.803)}{\frac{7}{8}a} \\
T_{\nu_e c} &= \frac{T_N}{R} \frac{\int_{-\infty}^{\alpha} d\alpha' \left(R' \frac{T}{T_N}\right)^4 W_\nu(\alpha';\alpha)}{\int_{-\infty}^{\alpha} d\alpha' \left(R' \frac{T}{T_N}\right)^3 W_\nu(\alpha';\alpha)}
\end{aligned}
$$

The integrands for all the conversion rates are all proportional to $p_{\nu_e}^2 dp_{\nu_e}$, as with n_{ν_e}. So for f_{ν_e} (rescaled p) in these rates use

$$
f_{\nu_e}(p) = \frac{n_{\nu_e}}{n_{\nu_0}(T_{\nu_e c})} \frac{1}{e^{\frac{p}{R}\frac{T_N}{T_{\nu_e c}}} + 1} \tag{4}
$$

Plugged into the integration for n_{ν_e} the above equation would give the correct expression for n_{ν_e} with neutrino heating.