The time derivative of f_ν obeys the Boltzmann Transport Equation. Designating $f_\nu(p)$ as 1 in the reaction 1+2 $\rightarrow$ 3+4

$$
\begin{aligned}
\frac{df_1}{dt} &= C_{coll} \qquad (1)\\
C_{coll} &= \frac{1}{2E_1}\left(\frac{kT_N}{R}\right)^5 \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} S|M|^2 \\
&\quad (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)\Lambda(f_1,f_2,f_3,f_4)\\
\Lambda(f_1,f_2,f_3,f_4) &= \{[1-f_1(p_1)][1-f_2(p_2)]f_3(p_3)f_4(p_4)\\
&\quad -f_1(p_1)f_2(p_2)[1-f_3(p_3)][1-f_4(p_4)]\}
\end{aligned}
$$

The first part of the phase space factor Λ corresponds to the reverse rate. The second part to the forward rate. One can solve this equation for a discrete set of values of p.

Tables (1) and (2) : Matrix Elements of the various reactions

The Matrix elements determine how a specific reaction alters f_{ν_e} and $f_{\nu_\mu} = f_{\nu_\tau}$. Actually, C_{coll} can be analytically integrated down to two variables p_2, p_3. As an example:

$$\begin{aligned}
\nu_e + \nu_\mu \quad &\leftrightarrow \quad \nu_e + \nu_\mu \\
C_{coll} \quad &= \quad \int_0^\infty dp_2 \int_0^{p_1+p_2} dp_3 F(p_1, p_2, p_3) \\
& \quad \{[1 - f_{\nu_e}(p_1)][1 - f_{\nu_\mu}(p_2)] f_{\nu_e}(p_3) f_{\nu_\mu}(p_1 + p_2 - p_3) \\
& \quad - f_{\nu_e}(p_1) f_{\nu_\mu}(p_2)[1 - f_{\nu_e}(p_3)][1 - f_{\nu_\mu}(p_1 + p_2 - p_3)]\}
\end{aligned}$$

$F(p_1, p_2, p_3)$ is a (very) complicated polynomial. C_{coll} could perhaps be solved implicitly, as you would solve for the isotope abundances Y_i. Then evolve the bins of $f_{\nu_e}(p_i)$ and $f_{\nu_\mu}(p_i)$ using 2nd. order Runge-Kutta, and plug them into the equations.