Neutron Diffusion and Nucleosynthesis in an Inhomogeneous Big Bang Model

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Abstract

This paper describes the evolution of a model of the universe with an inhomogeneous baryon distribution during the epoch of nucleosynthesis. The model is a lattice of spheres. Each sphere has a radius r_i known as the distance scale, and consists of a core and 63 spherical shells. The innermost shells have a high baryon number density while the outer shells have a low baryon number density. As the temperature of the model falls neutrons diffuse from the inner shells to the outer shells until the neutron number density has been homogenized. Subsequently at a temperature around 0.9 GK nucleosynthesis occurs, earlier in the high density shells than the low density shells. Neutrons in the inner shells are depleted to form ^{4}He nuclei; neutrons in the outer shells diffuse back to the inner shells and are also used to form ^{4}He nuclei. The distance scale is a measure of the scale of inhomogeneity in this model. This article will show how this inhomogeneity scale determines the fraction of neutrons that diffuses from the inner shells to the outer shells and then back, the fraction of protons indirectly redistributed by neutron diffusion, and ultimately the total production of ^{4}He.

1 Introduction

I have been working on a Big Bang Nucleosynthesis code for a standard (homogeneous and isotropic) model of the universe. This standard code (called the Texas code) can calculate the abundances of 68 different isotopes, the heaviest being ^{24}Ne. This code also has a section that calculates the neutrino heating effect on the abundances. I've worked these abilities into the Texas code in anticipation of applying them to a code for a model that has an inhomogeneous baryon distribution.

In February, 1999 I wrote an inhomogeneous BBN code, with the 68-isotope network and the neutrino heating section worked in the coding. In order to understand my IBBN code I had first to understand how the baryon distribution evolves with decreasing temperature in the inhomogeneous model.

At any given temperature the majority of baryons are in the form of free neutrons, free protons and ^{4}He nuclei. This paper covers the behavior of the number densities of these particular isotopes as they depend on the temperature of the model and on the parameters of the inhomogeneity.

2 Nucleosynthesis

My code starts at a high temperature $T = 100$ GK. At this temperature most baryons are in the form of free neutrons and free protons. The neutrons and protons interconvert between one another via the following weak reactions.

$$
\begin{aligned}
n + \nu_e &\leftrightarrow p + e^- \\
n + e^+ &\leftrightarrow p + \bar{\nu}_e \\
n &\leftrightarrow p + e^- + \bar{\nu}_e
\end{aligned}
$$

These reactions go much faster than the expansion rate of the universe ($\dot{\alpha}$), and so the reverse directions of the reactions can run as quickly as the forward directions. When the temperature drops to about 13 GK the rates of the first two reactions fall below the expansion rate. Subsequently among these weak reactions only neutron decay has a significant effect on the neutron and protonnumber densities.

Nuclear reactions such as

$$
\begin{aligned}
n + p &\leftrightarrow d + \gamma \\
p + d &\leftrightarrow {}^3He + \gamma \\
n + d &\leftrightarrow t + \gamma \\
p + t &\leftrightarrow {}^4He + \gamma \\
n +^3 He &\leftrightarrow {}^4He + \gamma
\end{aligned}
$$

are strong interactions and their rates are much faster than the weak interaction rates. At temperatures ≥ 1.0 GK their rates exceed the cosmic expansion rate, and they are in equilibrium with both reverse and forward reactions. At $T = 0.9$ GK the rate for reaction n + p $\leftrightarrow$ d + γ falls below the expansion rate. Neutrons and protons then form deuterium nuclei in significant numbers. The deuterium is subsequently used in reactions to form heavier nuclei, and then those nuclei are used to form even heavier nuclei. In a homogeneous and isotropic universe the final distribution of nucleons is around 75 % free protons, 25 % ^{4}He nuclei and traces of all other nuclei.

3 The Texas Inhomogeneous Model

The baryon distribution in my model is a lattice of spheres each with the same radius r_i, known as the distance scale. Each one sphere is characteristic of the whole universe. This sphere consists of a core and 64 spherical shells. I set a boundary $r_b = (1/2)r_i$. The 32 shells with radii less than r_b initially have a high baryon number density while the 32 outer shells have a low baryon number density (specifically 800 times lower). Figure (1) shows the total baryon number density, n_b, as a function of radius r, where r is normalized to run from 0.0 to 64.0. The graph shows neutron and proton number densities, n_n and n_p as well. The squares mark the central radius of each shell, and show that the shells near r_b have much smaller thicknesses than the shells far away from r_b.

$n(i, s)$ is the number density corresponding to isotope species i and shell s. The time derivative of $n(i, s)$ obeys the following equation.

$$\frac{dn(i,s)}{dt} = \frac{1}{n_b(s)} \sum_{j,k,l} N_i \left(-\frac{n^{N_i}(i,s) n^{N_j}(j,s)}{N_i! N_j!}[ij] + \right. \tag{1}$$

$$\left. \frac{n^{N_k}(k,s) n^{N_l}(l,s)}{N_k! N_l!}[kl] \right) \tag{2}$$

$$-3\dot{\alpha} n(i,s) + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_n \frac{\partial n(i,s)}{\partial r} \right) \tag{3}$$

$n_b(s)$ is the total baryon number density in shell s. The first two terms corresponds to nuclear reactions within shell s of the form

$$N_i \left({}^{A_i} Z_i \right) + N_j \left({}^{A_j} Z_j \right) \leftrightarrow N_k \left({}^{A_k} Z_k \right) + N_l \left({}^{A_l} Z_l \right)$$

The first term accounts for the forward direction of this reaction, which destroys isotopes i and j and creates isotopes k and l. The second term accounts for the reverse direction, which creates isotope i. The third term corresponds to the expansion of the universe. And the final term corresponds to diffusion of isotope i into and out of shell s. For my model only the neutrons diffuse from shell to shell. The diffusion coefficient D_n accounts for collisions between neutrons and electrons and neutrons and protons that occur as the neutrons diffuse.

4 Results

Figure (2) is a contour map of the overall final mass fraction $X_{^4\mathrm{He}}$ of ^{4}He. $X_{^4\mathrm{He}}$ depends on the baryon-to-photon ratio η and the inhomogeneity scale which I parametrize as the distance scale r_i in cm at the time when $T = 100$ GK. For this paper I set η to 3 $\times 10^{-10}$ and then discuss the behavior of neutrons, protons and ^{4}He nuclei for selected values of r_i. Table (1) below lists $X_{^4\mathrm{He}}$ for each r_i.

r_i in cm.	$X_{^4\mathrm{He}}$
10^2	0.239029657
1.58489×10^3	0.239369667
2.51189×10^4	0.243663794
10^5	0.241847338
7.94328×10^5	0.229781933
6.30957×10^6	0.250711334

Table (1): ^{4}He when $\eta = 3.0 \times 10^{-10}$.

4.1 r_i = 100 cm

Figure (3) shows the number densities n_n, n_p and 4 $\times n_{^4\mathrm{He}}$ for the case $r_i = 100$ cm. r_i is so small that neutrons diffuse from the high density shells to the low density shells rapidly compared to the expansion rate. By $T = 60$ GK the neutrons have become homogeneously distributed over the shells. During these high temperatures the neutron/proton conversion rates run much faster than the expansion rate. The reactions that convert protons into neutrons can keep up with the diffusion. Protons in the high density shells are converted to neutrons which then diffuse out to the low density regions. So the proton number density gets homogenized as well. For T less than 60 GK the model behaves like a homogeneous model, and yields a homogeneous value of $X_{^4\mathrm{He}}$ of 0.23903.

4.2 r_i = 1600 cm

If $r_i = 1600$ cm neutron diffusion peaks at around $T = 20$ GK and homogenizes the neutron distribution by $T = 10$ GK. At this temperature range the conversion rates are not as high as for the case of $r_i = 100$ cm. Figure (4) shows that for this value of r_i many of the protons are redistributed but the high density shells still retain more protons than the low density shells. So the inner shells still have a higher baryon density when nucleosynthesis begins. Figure (5a) shows for the inner shell 3 the diffusion rates into and out of shell 3 and the conversion rates within shell 3, with all the rates normalized to the expansion rate of the universe. As expected the diffusion rate out of shell 3 into shell 4 is faster than the rate out of shell 2 to shell 3. The neutron depletion makes the p $\rightarrow$ n rates higher than the n $\rightarrow$ p rates, but at a temperature near to when the neutrons and protons fall out of equilibrium. Figure (6a) shows the same rates for the outer shell 62. The diffusion rate from shell 61 into shell 62 is the faster rate, as are the n $\rightarrow$ p rates in this shell.

Because of the higher density nucleosynthesis occurs slightly earlier in the inner shells than the outer shells. Nucleosynthesis depletes the free neutrons more quickly in the inner shells. Figure (5b) shows the forward and reverse rates of n + p $\leftrightarrow$ d + γ for shell 3, and Figure (6b) shows the corresponding rates for shell 62. The graphs also show diffusion rates for these shells. Because neutrons are used up earlier in the inner shells neutrons from the other shells will diffuse back into the inner shells. Figure (5b) shows the diffusion rate from shell 4 into shell 3 faster than the rate from shell 3 out to shell 2, and the rates peaking at the temperature when n + p $\leftrightarrow$ d + γ falls out of equilibrium. With earlier nucleosynthesis and neutrons flowing back in the inner shells have a great production of ^{4}He than the outer shells, as shown in Figure (4). The code yields an overall mass fraction of ^{4}He of 0.23937, higher than for $r_i = 100$ cm.

4.3 $r_i = 25000$ cm

Figures (7a) and (7b) show n_n, n_p and $4 \times n_{^4\mathrm{He}}$ from both sides of the shell radius axis so that one can see the region where n_n is homogeneous. Diffusion does not have a significant effect on n_n until when T = around 8 GK and n_n isn't homogenized until $T = 3$ GK. By then the conversion rates have fallen out of equilibrium. In the high density inner shells the conversion reactions can't convert protons to neutrons during diffusion. So these shells retain nearly all their protons, as shown in the graph for n_p in Figure (7b). The large flow of neutrons into the low density outer shells increases the n $\rightarrow$ p conversion rates, but not enough to produce as many protons as there are neutrons in these shells.

With no proton depletion the inner shells have the highest baryon densities they could have for models where the neutron diffusion can be homogenized. Nucleosynthesis in the inner shells occurs earlier than for smaller values of r_i. Neutrons in the outer shells will diffuse back into the inner shells, as they did for $r_i = 1600$ cm. In fact nearly all outer shell neutrons will diffuse to the inner shells, and all ^{4}He production will come from the inner shells. Figure (7a) shows this as the graph for n_p for the outer shells shows no depletion due to nucleosynthesis. The inner shells produce an overall final mass fraction $X_{^4\mathrm{He}}$ of 0.24366, larger than for any smaller r_i.

4.4 $r_i = 10^5$ cm

$r_i = 25000$ cm is the smallest distance scale wherein the high density inner shells can retain all their protons during diffusion, and the largest distance scale wherein neutrons can reach all inner shells through back diffusion during nucleosynthesis. The mass fraction of ^{4}He for this distance scale turns out to be a maximum value in Figure (2) along the line for $\eta = 3 \times 10^{-10}$. Figure (8) shows the number densities for a larger distance scale, $r_i = 10^5$ cm. Diffusion becomes significant in the graph for n_n when $T = 3.0$ GK, and n_n becomes homogeneous around $T = 1.5$ GK. Figures (9a) and (9b) show the diffusion rates in shell 3 for this distance scale, and Figures (10a) and (10b) show the rates in shell 62. The peaks of the diffusion rates in (9a) and (10a) agree with the behavior of n_n in Figure (8), and the diffusion rates are higher when the neutron flow switches to back diffusion than for smaller distance scales. Through back diffusion the neutrons reach the inner shells near r_b, but nucleosynthesis has ended for the core and innermost shells by the time the neutrons back diffuse to them. In Figure (8) the graph for $4 \times n_{^4\mathrm{He}}$ has a peak near r_b but decreases with decreasing radius. The final overall mass fraction of ^{4}He for this distance scale is 0.24185, smaller than for $r_i = 25000$ cm.

4.5 $r_i = 7.9 \times 10^5$ cm

$X_{^4\mathrm{He}}$ continues to decrease with increasing r_i until the distance scale reaches 7.9 $\times 10^5$ cm. Figure (11) shows the number densities for this distance scale. For the innermost shells, nucleosynthesis occurs before the neutron distribution can be homogenized by diffusion. Figure (12a) shows that in shell (3) the diffusion rates reach their peak values just when nucleosynthesis starts. When nucleosynthesis starts the inner shells have a baryon density larger than they have for smaller values of r_i because the shells still have neutrons that were lost through diffusion in the preceding models. Nucleosynthesis occurs earlier and more rapidly than in the preceding models, as shown in the steeper drop of the rates for n + p $\leftrightarrow$ d + γ in Figure (12b). In Figure (13a) nucleosynthesis also occurs right when the diffusion rates peak in shell 62. Diffusion brings some neutrons in raising the conversion rates, but the conversion rates don't reach the levels of the cases where the neutron distribution has been fully homogenized. With fewer neutrons shell 62 has a lower baryon density than before, and nucleosynthesis is slower as shown in the rates for n + p $\leftrightarrow$ d + γ in Figure (13b). The inner shells deplete their neutrons more rapidly than the outer shells, leading to the large back diffusion in Figure (12b) that causes the rate for n + p $\rightarrow$ d + γ to increase briefly. In Figure (11), the graph for 4 $\times n_{^4\mathrm{He}}$ has a sharper peak near r_b than before and then a decreasing value with decreasing shell radius. But the graph increases for the innermost shells reflecting the larger baryon densities these shells have when nucleosynthesis occurs there. The final value of $X_{^4\mathrm{He}}$, 0.22978, is the lowest value for $\eta = 3.0 \times 10^{-10}$

4.6 $r_i = 6.3 \times 10^6$ cm

The mass fraction for ^{4}He increases for distance scales larger than 7.9 $\times 10^5$ cm. Figure (14) shows n_n, n_p and 4 $\times n_{^4\mathrm{He}}$ for distance scale $r_i = 6.3 \times 10^6$ cm. The final value of $X_{^4\mathrm{He}}$ is 0.25071 for this model. The sizes of the shells are too large for the neutrons to undergo any significant diffusion before the end of nucleosynthesis, as shown in the figure for the inner shells. The high density inner shells and the low density outer shells behave as two separate standard model codes where the final mass fractions are the weighted averages of the two models. The large baryon number density of the inner shells leads to a final mass fraction of ^{4}He considerably larger than the standard model value for $X_{^4\mathrm{He}}$.

Figures (15) and (16) summarize the results for the six distance scales discussed in this talk. Figure (15) shows the neutron number density n_n for both shell 3 and shell 62, as a function of temperature T. One can see on the graph when diffusion homogenizes the neutron distribution for r_i less than 7.9 $\times 10^5$ cm, and when neutrons are depleted by nucleosynthesis. The point where the graphs for n_n of each shell comes together occurs later with increasing r_i. After nucleosynthesis the graphs diverge from each other slightly, reflecting the different times when nucleosynthesis is occuring in each shell. The graphs for $r_i = 7.9 \times 10^5$ cm don't come together. The graph for n_n in shell 3 is considerably steeper during nucleosynthesis than the graph for shell 62. Figure (16) shows the graphs for 4 $\times n_{^4\mathrm{He}}$ at the end of the runs and as functions of shell radius. This figure shows the flat line for $r_i = 100$ cm that corresponds to a standard model result. The graph gets larger for the inner shells when $r_i = 1600$ cm. For $r_i = 25000$ cm $n_{^4\mathrm{He}}$ has a plateau for the inner shells reflecting the fact that the inner shells now maintain all their protons for nucleosynthesis. The graph dips for the inner shells and rises slightly for the outer shells for $r_i = 10^5$ cm indicating that back diffusion cannot distribute all the neutrons back to the inner shells before nucleosynthesis ends. Then for $r_i = 7.9 \times 10^5$ cm nucleosynthesis begins in the innermost shells before the neutrons there diffuse out, and $n_{^4\mathrm{He}}$ rises again for the innermost shells. At $r_i = 6.3 \times 10^6$ cm the graph has plateaued again for the inner shells and diffusion affects the graph only for the shells near r_b.

5 Future Research

The authors have presented results for ^{4}He production in an IBBN code for spherically symmetric models with a high density core and a low density outer layer, for a specified boundary $r_b = 0.5r_i$ between the high and low density shells and with a specified density contrast $R_\rho = 800$. The code is easily adaptable to planar and cyllindrical symmetry, and the effect of these geometrical changes will be investigated.

The code can produce contour maps for all 68 isotopes; this will allow an investigation of how ^{9}Be, ^{11}B, ^{12}C and ^{16}O depend on the inhomogeneity of the universe. It would be exciting if substantial elevation of these abundances occured for some parameters. The neutrino heating effect has not yet been thoroughly investigated in inhomogeneous models; there may be interesting modifications of the predicted abundances. All these questions are under investigation.

6 References

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