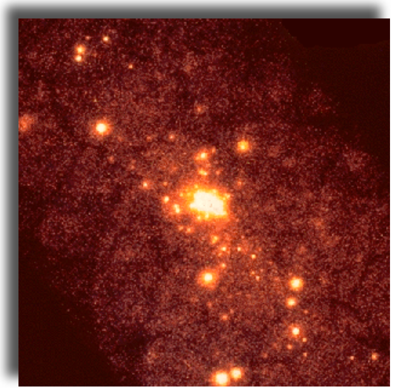
# Simplicity of Binary Black Hole Coalescence and its Implications for Detection

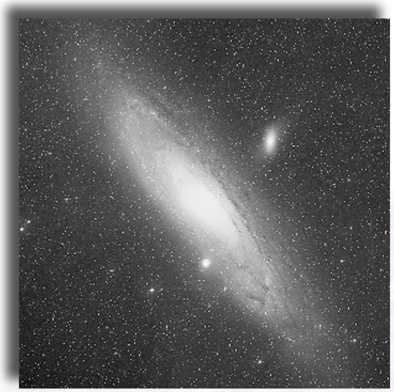
Deirdre Shoemaker
Center for Relativistic Astrophysics
School of Physics
Georgia Institute of Technology

# The Universe Today

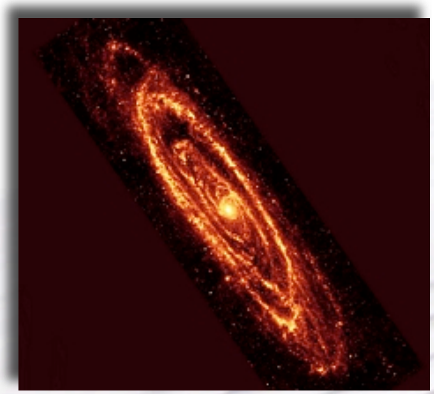


X-ray

UV

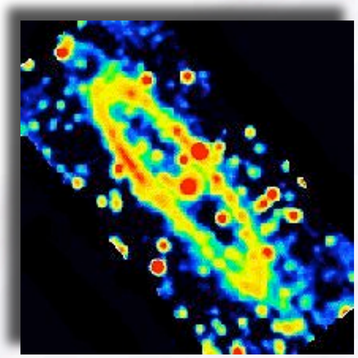


Optical



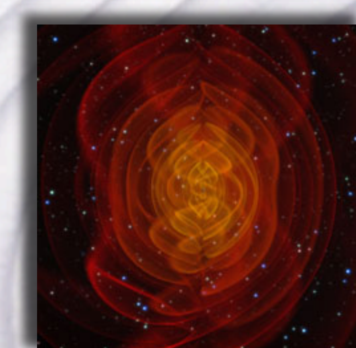
mid-IR

far-IR



Radio

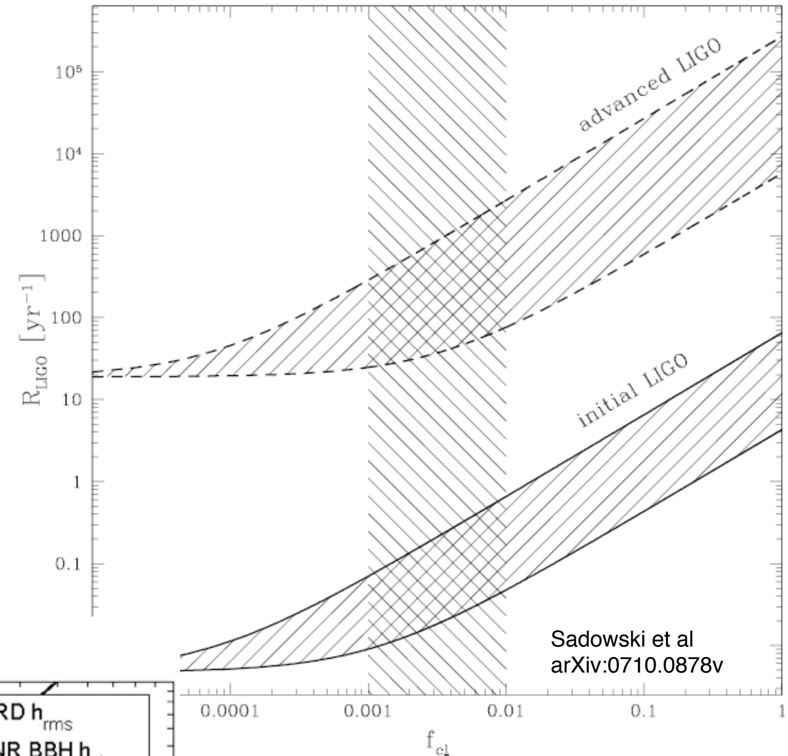
Neutrinos &
Cosmic Rays

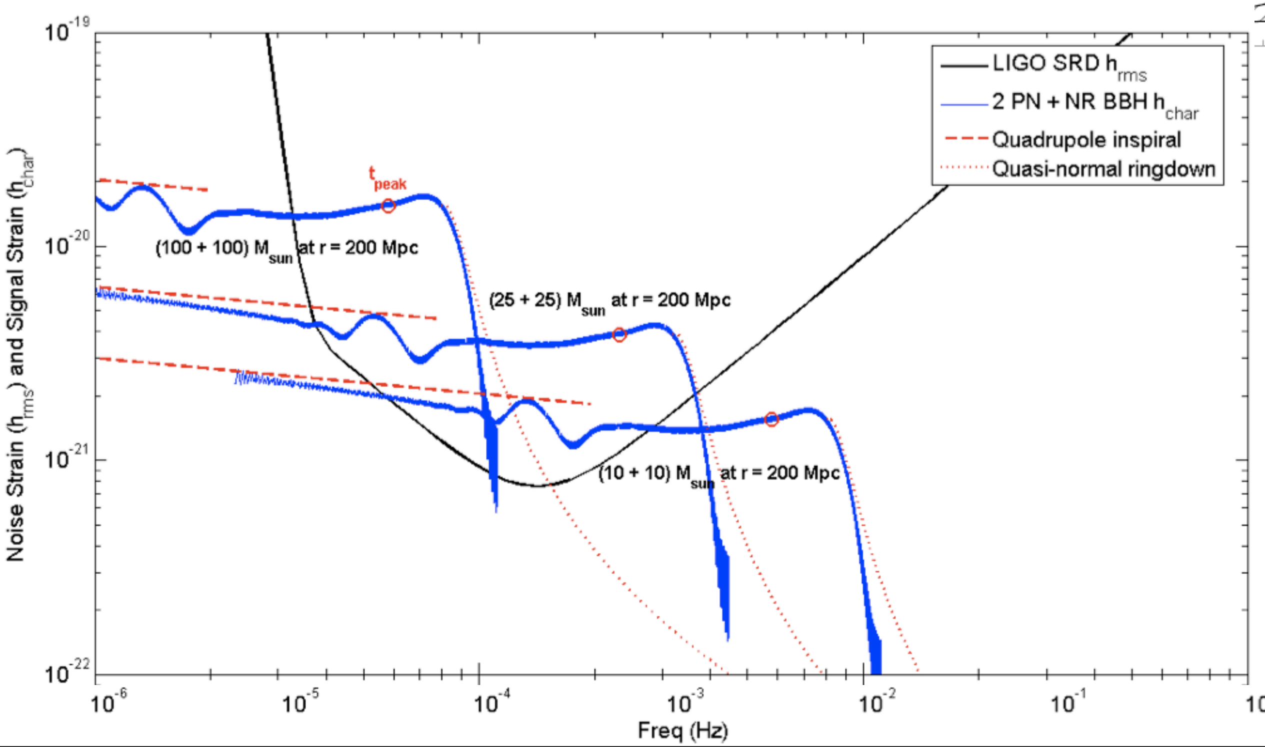


Gravitational
Waves

# Gravitational Waves: LIGO

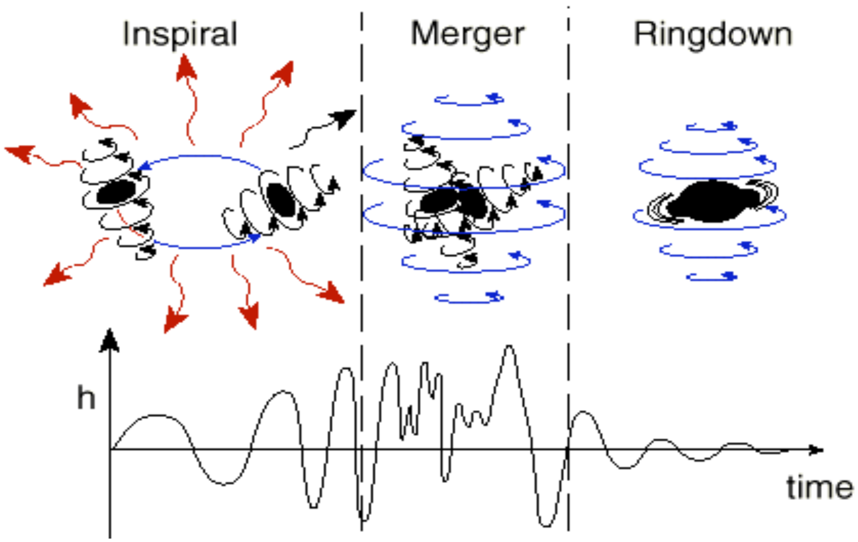
BBH Initial LIGO: < .1/year
Advanced LIGO: > 10/year
Low signal-to-noise signals
BBH Signal lasts milli-seconds
Waveforms necessary for detection



# Binary Black Hole Coalescence

- Numerical Relativity
  - extreme gravity, where no approximations currently hold
  - solves the Einstein equation for dynamical spacetime
  - requires computational and theoretical innovations
- Goals are to
  - unveil gravity in its strongest regime
  - inform gravitational wave detection
  - determine characteristics of final black hole
- BBH Stages
  - inspiral
  - merger
  - ringdown
- Major Breakthroughs 2005
  Pretorius, RIT Team, GSFC Team




K. Thorne

# Collaboration ...

arXiv:0807.3292 [ps, pdf, other]

**Superkicks in Hyperbolic Encounters of Binary Black Holes**

James Healy, Frank Herrmann, Ian Hinder, Deirdre M. Shoemaker, Pablo Laguna, Richard A. Matzner

Comments: 4 pages, 5 figures, 1 table

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Astrophysics (astro-ph)

arXiv:0802.2520 [ps, pdf, other]

**Binary Black Hole Encounters, Gravitational Bursts and Maximum Final Spin**

M. C. Washik, J. Healy, F. Herrmann, I. Hinder, D. M. Shoemaker, P. Laguna, R. A. Matzner

Comments: Replaced with version to appear in PRL

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Astrophysics (astro-ph)

arXiv:0706.2541 [ps, pdf, other]

**Binary Black Holes: Spin Dynamics and Gravitational Recoil**

Frank Herrmann, Ian Hinder, Deirdre M. Shoemaker, Pablo Laguna, Richard A. Matzner

Comments: 15 pages, 10 figures, replaced with version accepted for publication in PRD

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Astrophysics (astro-ph)

arXiv:gr-qc/0701143 [ps, pdf, other]

**Gravitational recoil from spinning binary black hole mergers**

Frank Herrmann, Ian Hinder, Deirdre Shoemaker, Pablo Laguna, Richard A. Matzner

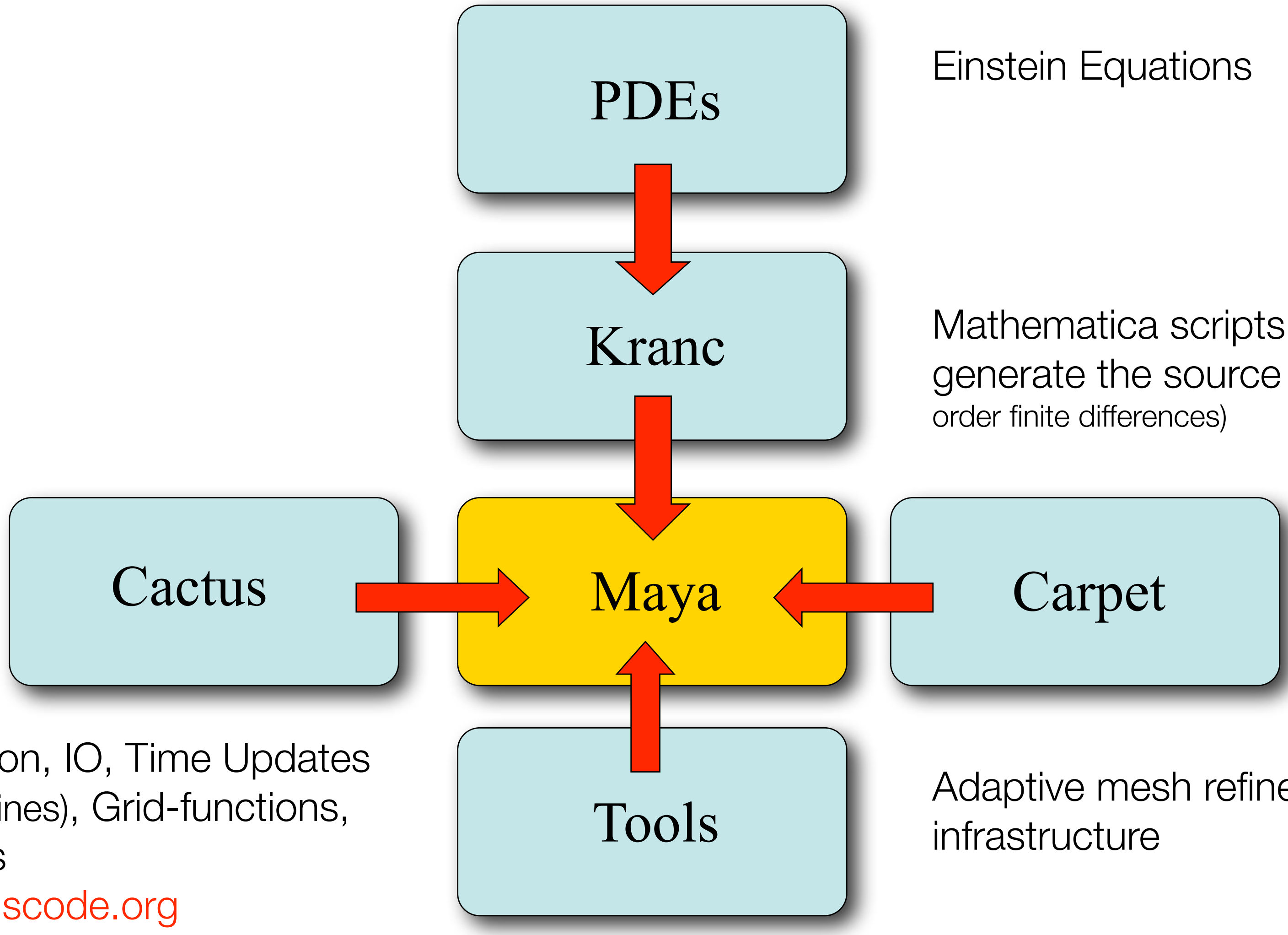
Comments: 8 pages, 8 figures, replaced with version accepted for publication in ApJ

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Astrophysics (astro-ph)

# Binary Black Holes in Eccentric Orbits

- Eccentric Binaries in the non-linear phase
- Comparing with post-Newtonian
  [Hinder, Herrmann, Laguna, Shoemaker arXiv:0806:1037 (2008)]
- How does the binary approach its final state?
  [Hinder, Vaishnav, Herrmann, Laguna, Shoemaker PRD 77, 081502 (2008)]
- Are ground-based detectors blind to eccentric, intermediate mass black hole binaries?
  [Vaishnav, Hinder, Herrmann, Shoemaker in preparation]

# The MayaKranc Code




Einstein Equations

Mathematica scripts to generate the source (6th order finite differences)

Parallelization, IO, Time Updates (method of lines), Grid-functions, Parameters
www.cactuscode.org

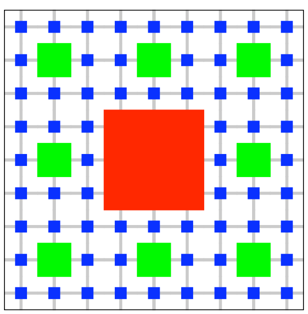
Adaptive mesh refinements infrastructure

Horizon trackers, wave extraction, etc.

# Binary Black Holes in Eccentric Orbits

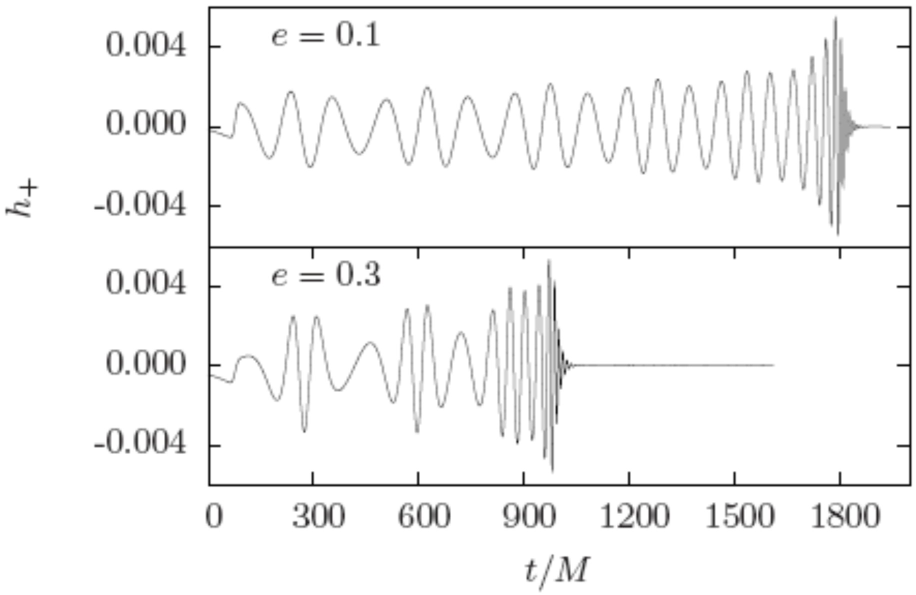
Ranger, TACC

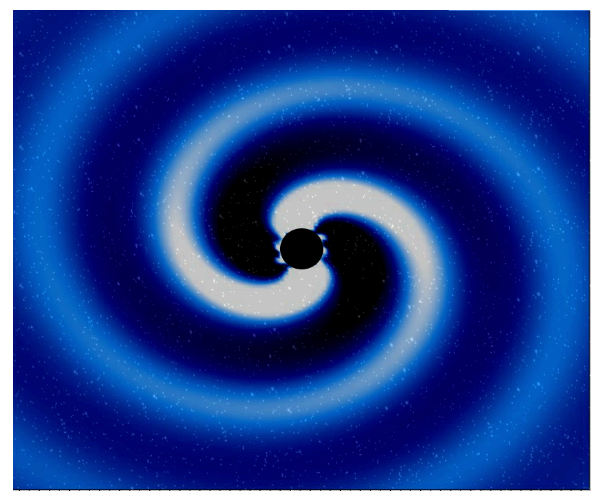
```
dot[A[li,lj]] -> em4phi B[li,lj] - em4phi/3 h[li,lj] trB
    + alpha K A[li,lj] - 2 alpha A[li,lk] A[lm,lj] hInv[um,uk]
    + beta[uk] PDonesided2nd[A[li,lj], lk] + 2 Symmetrize[A[lk,li]
    PD[beta[u
    k],lj],li,lj]
    - 2/3 A[li,lj] PD[beta[uk],lk] ,

divA[ui] -> -hInv[uk,ua] AInv[ub,ui] PD[h[la,lb],lk] - hInv[ua,ui] AInv[uk,ub]
    PD[h[la,lb
    ],lk]
    + hInv[uk,ul] hInv[ui,um] PD[A[ll,lm],lk],

dot[Gam[ui]] -> GamRHS
```



- Cactus & Carpet (Schnetter)
- Moving punctures approach (RIT & NASA)
- Kranc (Husa, Hinder, Lechner)
- 6th order finite differencing
- Initial Data: PN parameters in puncture approach and TwoPuncture Code (Ansorg)

# Gravitational Waveform



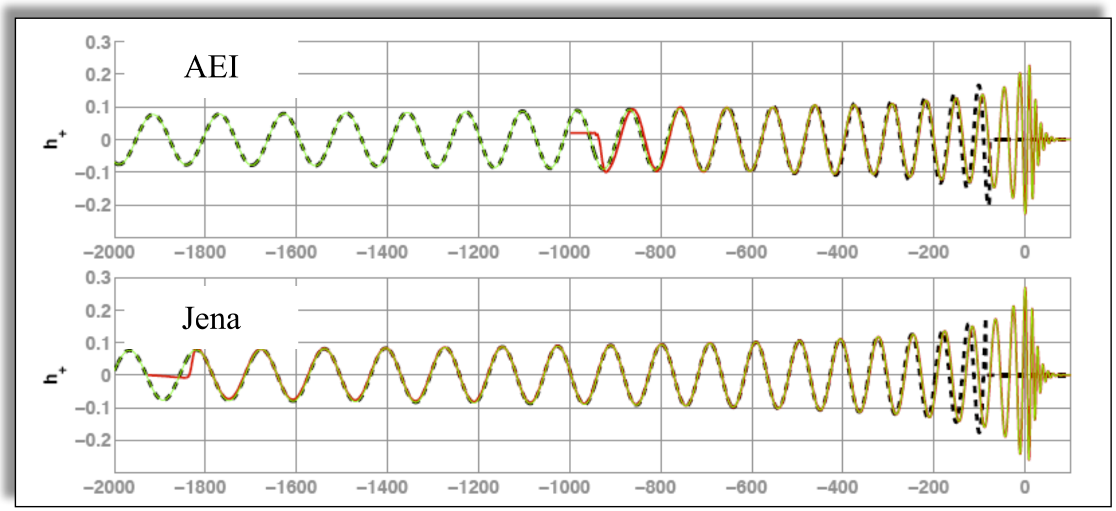
During the merger, at least 3% of the mass of the binary is converted to energy.

$$E = \epsilon 20 M_{\odot} c^2$$

$$L = \frac{E}{t} = 7 \times 10^{41} \mathrm{ergs/s}$$

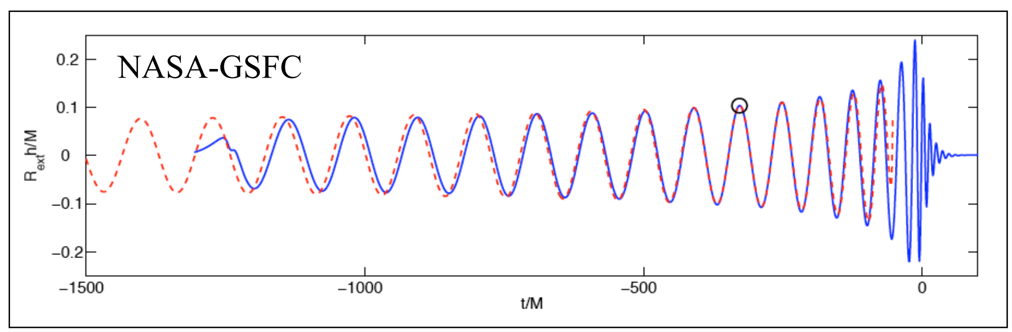
# Early Stage: Post-Newtonian

# Binary Black Hole Codes Today



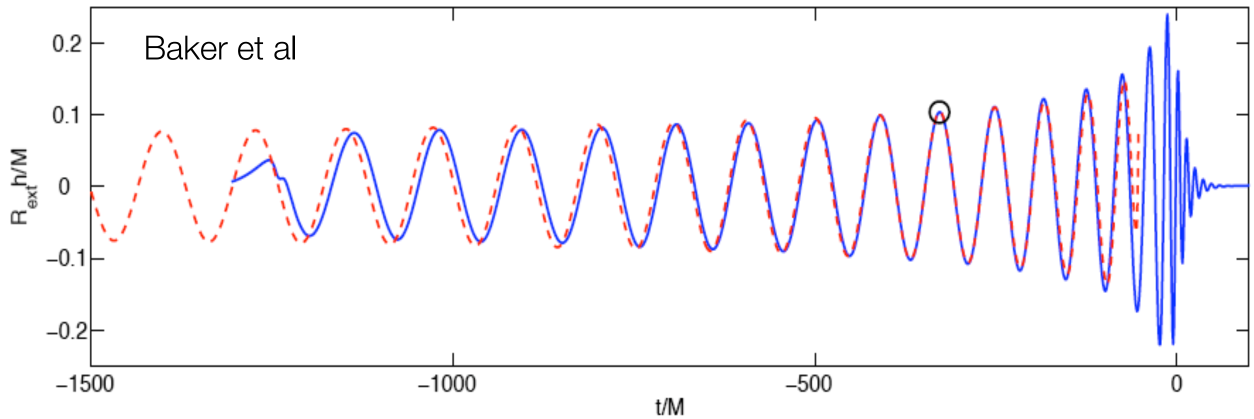

Comparisons with Post-Newtonian:
Finding the limits of validity!

Merger waveforms:
Making the most of LIGO data!



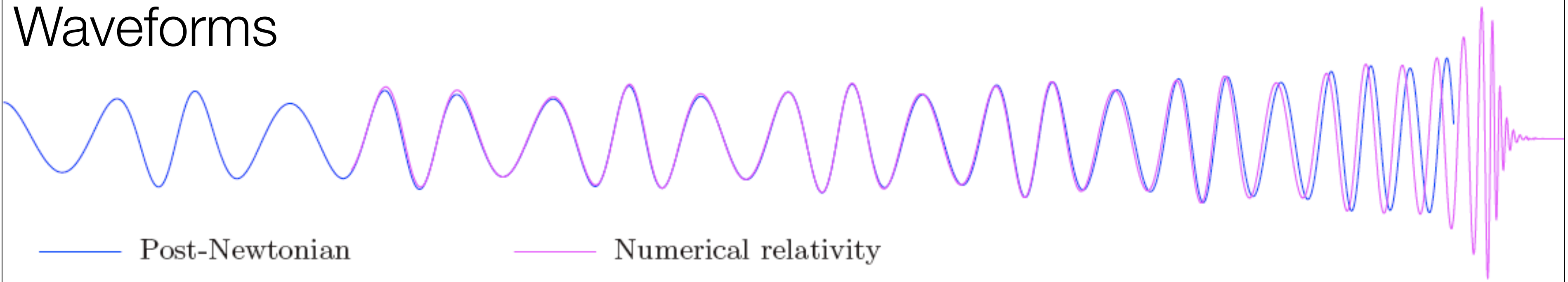
# NR and PN Comparisons

Circular Comparisons: equal-mass, non-spinning comparisons agree up to 2-3 orbits before merger.




- Baker et al (2006) and (2007)
- Buonanno et al (2006)
- Boyle et al (2007) and (2008)
- Berti et al (2007)
- Pan et al (2008)
- Hannam et al (2007) and (2008)
- Damour et al (2008)

# First Comparison between PN and NR for equal-mass, eccentric Waveforms




| Post-Newtonian | Numerical Relativity |
| --- | --- |
| 3PN conservative + 2PN radiation reaction $h_+, h_x(\dot{\phi}, \dot{e}, \dot{l}, \dot{x})$ | BSSN formalism $h_{ij}, \gamma, A_{ij}, K$ <br> $e \sim 0.1$ |
| $\Psi_4^{22} = \ddot{h}^{22}$ | $\Psi_4^{22}$ extracted |

$$\omega_{gw} \equiv \dot{\phi}_{gw} = \frac{d}{dt} \arg \Psi_4^{22}$$

Based on Ian Hinder et al arXiv:0806.1037

# PN Waveforms

3 PN conservative quasi-Keplerian expressions [Memmesheimer, Gopakumar and Schaefer, 2004]

$$x = ((2\pi + \Delta\phi)/P)^{2/3}$$

$$r/M = [1 - e_t \cos u]\, x^{-1} + r_{1\mathrm{PN}} + r_{2\mathrm{PN}} x + r_{3\mathrm{PN}} x^2 + \mathcal{O}\left(x^3\right)$$

$$M\dot{\varphi} = \frac{1\sqrt{1-e_t^2}}{[1 - e_t \cos u]^2} x^{3/2} + \dot{\varphi}_{1\mathrm{PN}} x^{5/2} + \dot{\varphi}_{2\mathrm{PN}} x^{7/2} + \dot{\varphi}_{3\mathrm{PN}} x^{9/2} + \mathcal{O}\left(x^{11/2}\right)$$

$$l = u - e_t \sin u + l_{2\mathrm{PN}} x^2 + l_{3\mathrm{PN}} x^3 + \mathcal{O}\left(x^4\right)$$

$$M\dot{l} = Mn = x^{3/2} + n_{1\mathrm{PN}} x^{5/2} + n_{2\mathrm{PN}} x^{7/2} + n_{3\mathrm{PN}} x^{9/2} + \mathcal{O}\left(x^{11/2}\right)$$

Supplemented by adiabatic 2 PN radiation reaction expressions [Koenigsdoerffer and Gopakumar, 2006]

$$M\dot{x} = \frac{2\eta}{15(1-e_t^2)^{7/2}}\left(96 + 292 e_t^2 + 37 e_t^4\right) x^5 + \dot{x}_{1\mathrm{PN}} x^6 + \dot{x}_{1.5\mathrm{PN}} x^{13/2} + \dot{x}_{2\mathrm{PN}} x^7 + \mathcal{O}\left(x^{15/2}\right)$$

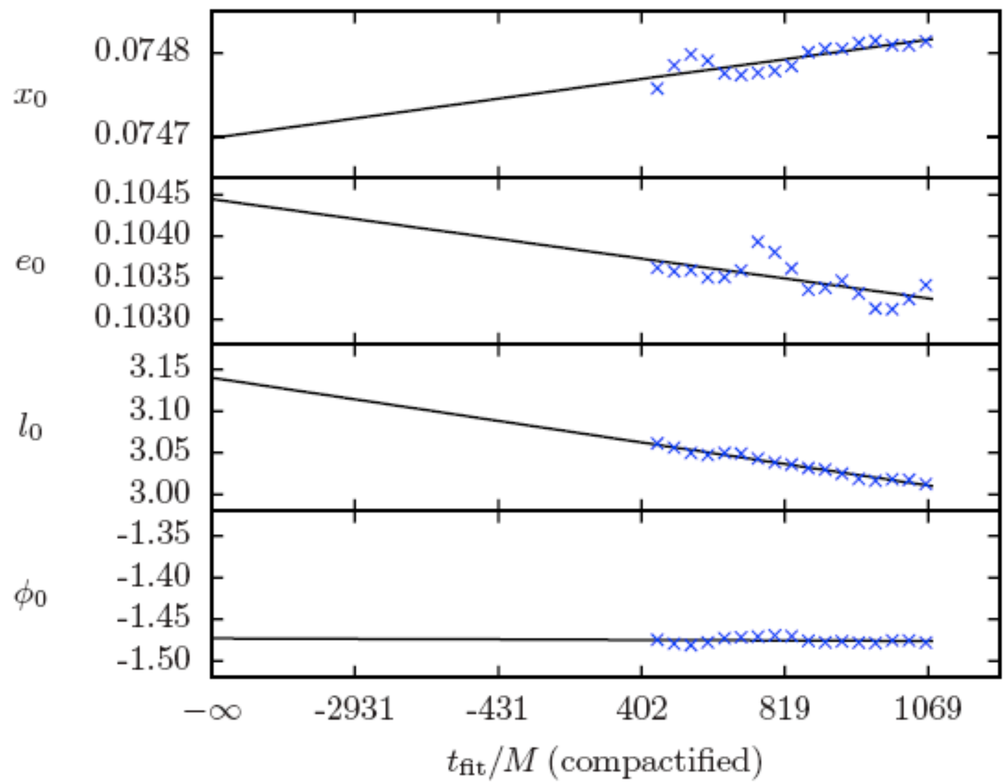
$$M\dot{e} = \frac{-e\eta}{15(1-e_t^2)^{5/2}}\left(304 + 121 e_t^2\right) x^4 + \dot{e}_{1\mathrm{PN}} x^5 + \dot{e}_{1.5\mathrm{PN}} x^{11/2} + \dot{e}_{2\mathrm{PN}} x^6 + \mathcal{O}\left(x^{13/2}\right)$$

We then construct restricted waveforms (Newtonian accurate in the amplitude) for the purpose of comparison with NR.

Likely require higher than 2 PN radiation reaction to match better towards end of simulation; circular case is matched using 3.5 PN.

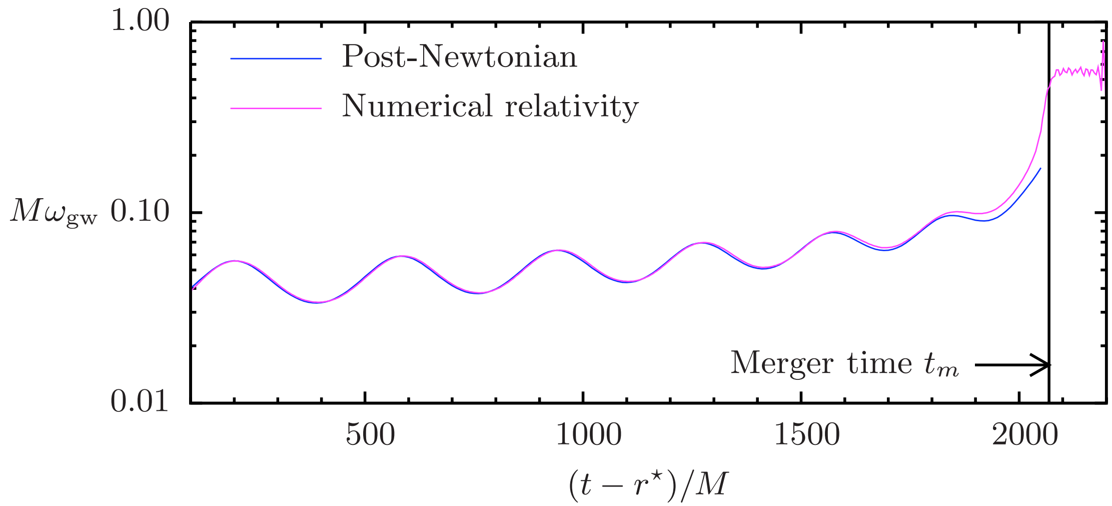
# Parameters to Fit

Nonlinear Least Squares fit of PN Parameters $y_0 \equiv [x_0, e_0, l_0, \phi_0]$
NR and PN match in the limit as $t \to -\infty$, extrapolate



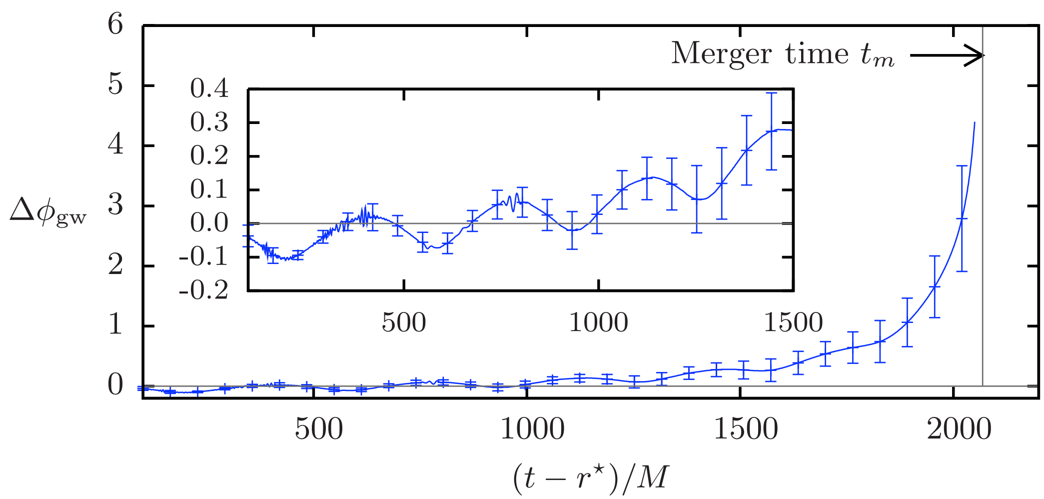

$t_{\rm fit}/M$ center of the fitting window

# Agreement between PN and NR




The difference in gravitational wave phase, $\phi_{gw}$ , between NR and PN is < 0.1 radians for the first 8 cycles (=1000M), and grows to ~0.8 radians at $M\omega_{gw} = 0.1$

$$\phi_{gw}(t - r^*) \equiv \arg\left[\Psi_4^{22}(t - r^*)\right]$$

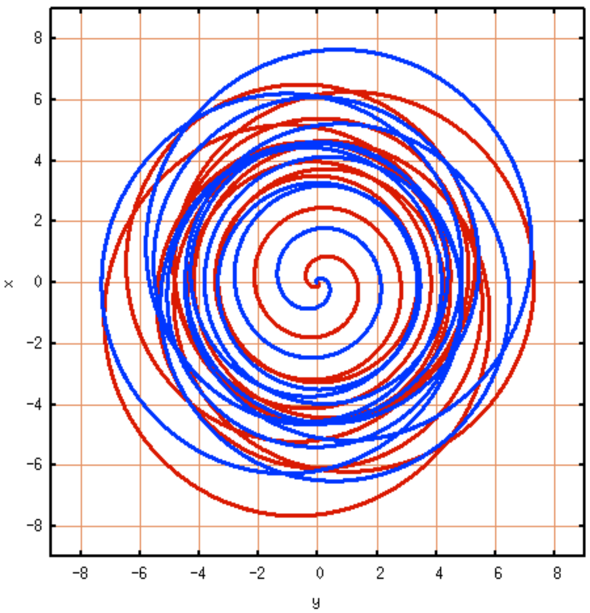



The difference between NR and PN of 0.8 radians could be due to the 2PN radiation reaction, comparisons for the circular case using only 2PN radiation reaction gave similar results.

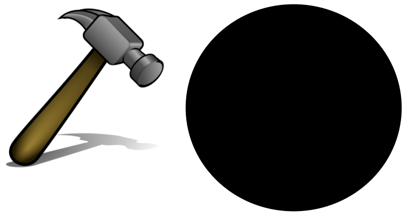
# Late Phase: The Ringdown

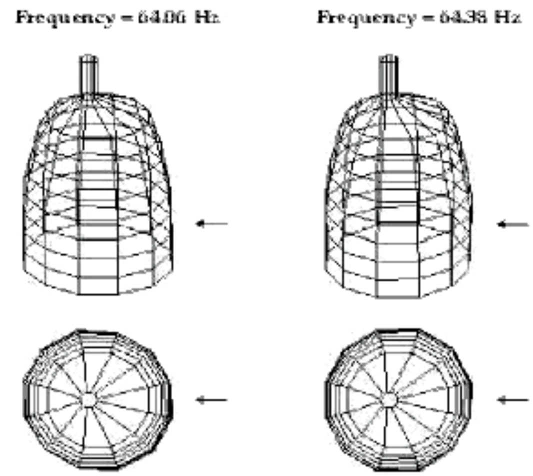
# The Approach to Kerr: How black holes loose their hair

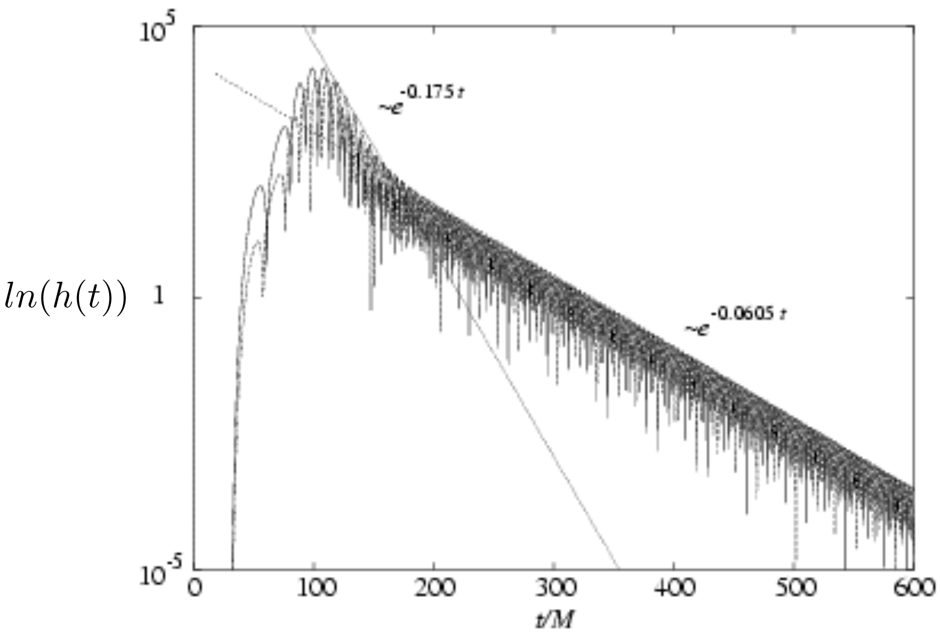
- Black Holes in isolation:
- mass (M), angular momentum (J) and charge (Q)
- Information Lost Through:
  - Radiation
  - Horizon (Hawking radiation)
- Can NR reveal how info gets radiated in last stages?



# A Perturbed BH Settles to Kerr by Ringing





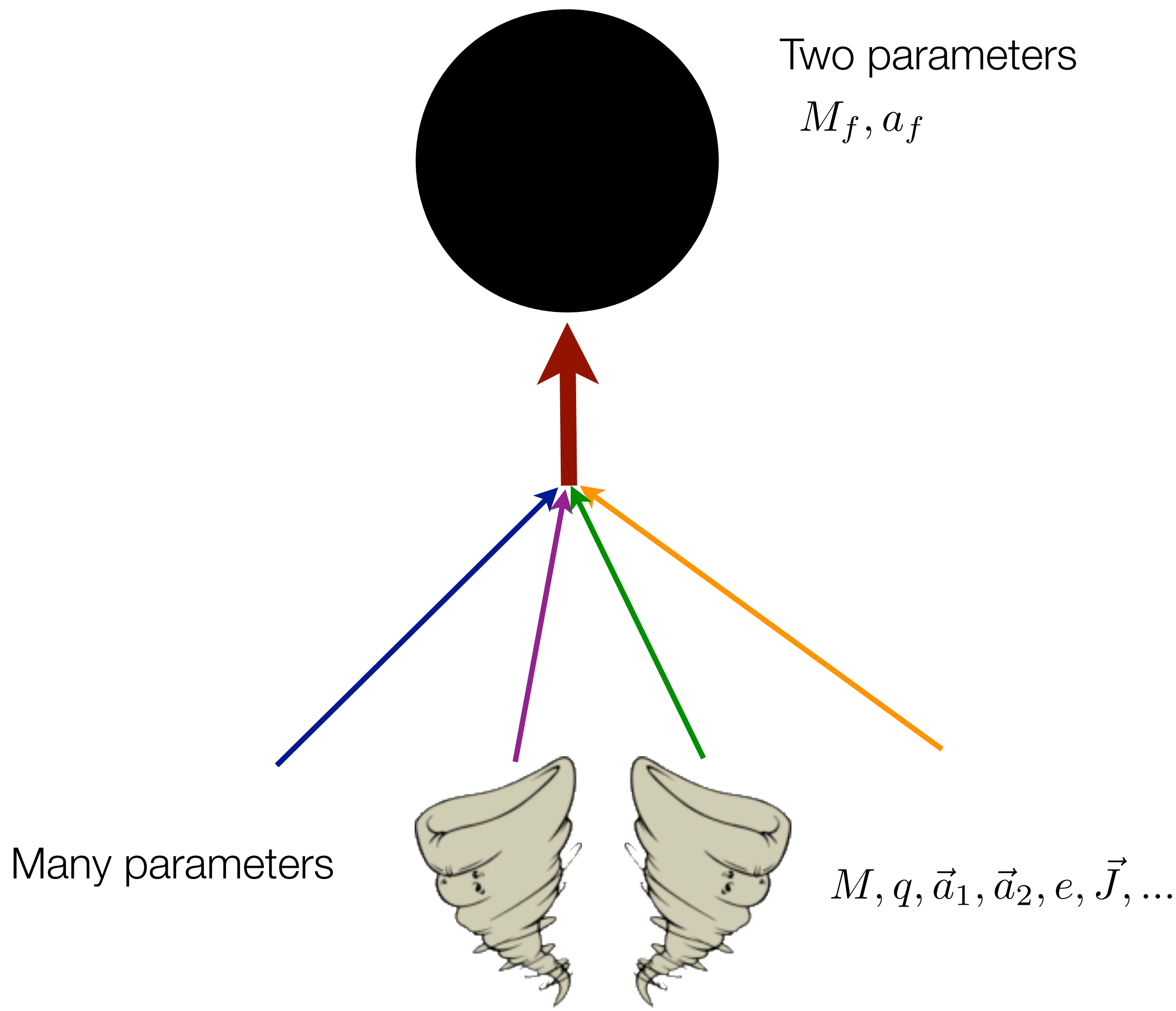

• Ringdown is completely describable by damped sinusoidal functions.

$$h(t) \propto e^{-t/\tau} sin(2\pi f t)$$

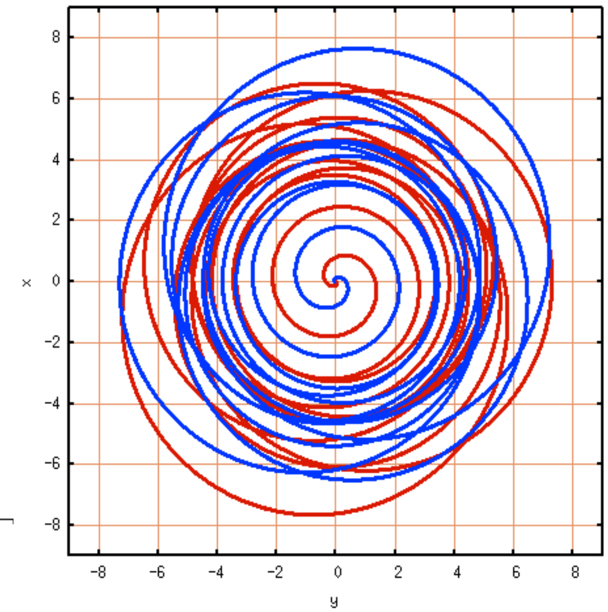
• The black hole “rings” in tones given by a set of unique complex frequencies.

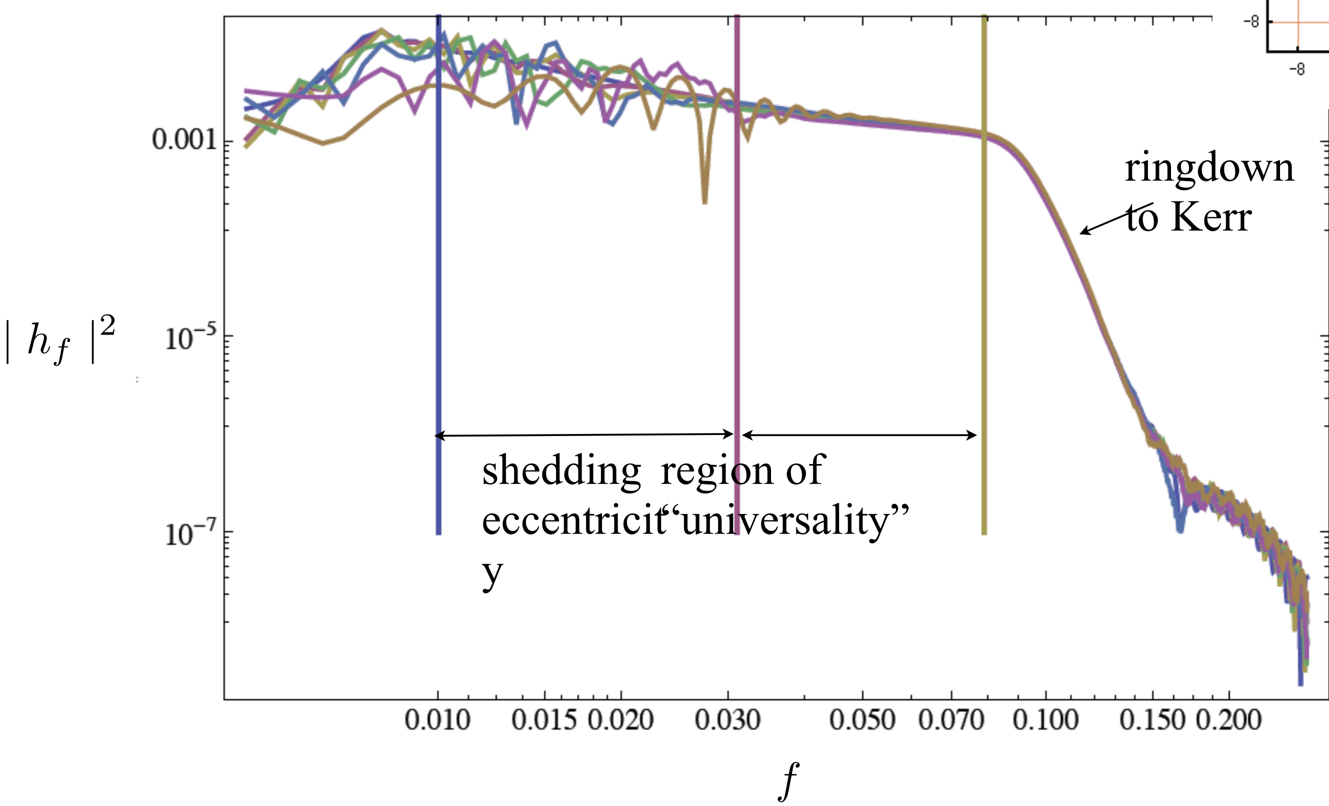
$$f(M, a) \quad \tau(a)$$

# Many ways to a Kerr Black Hole

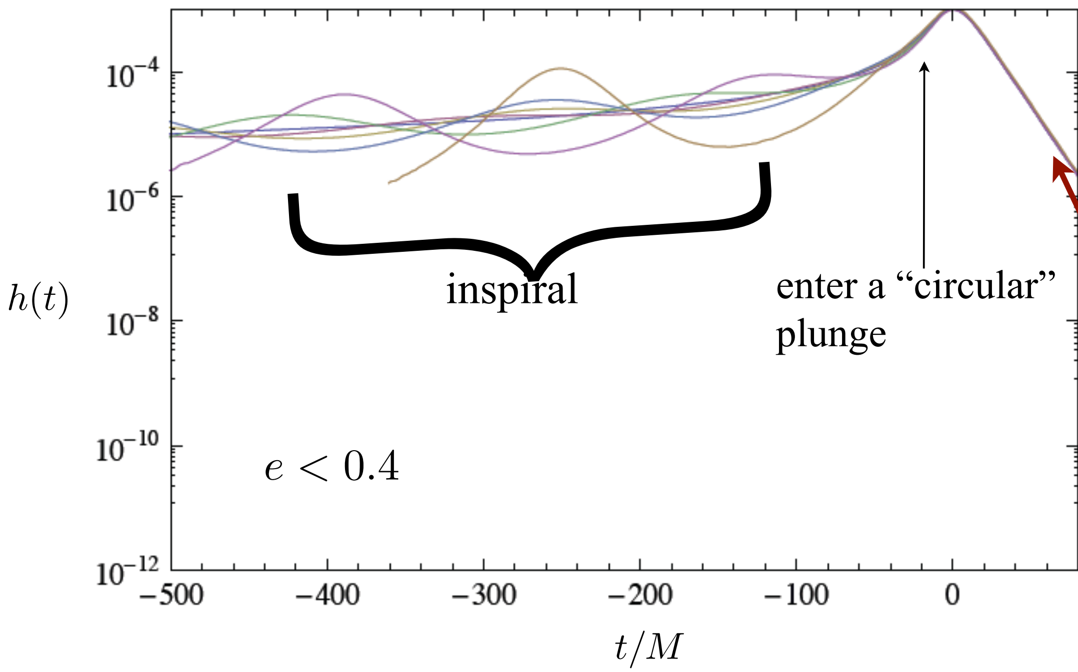


# Shedding of Hair in BBH Systems: eccentric case



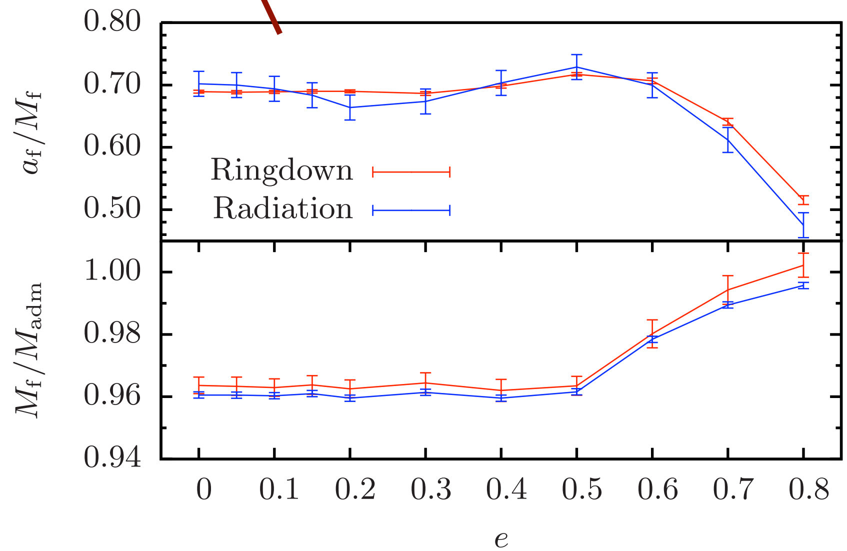



# Shedding Eccentricity



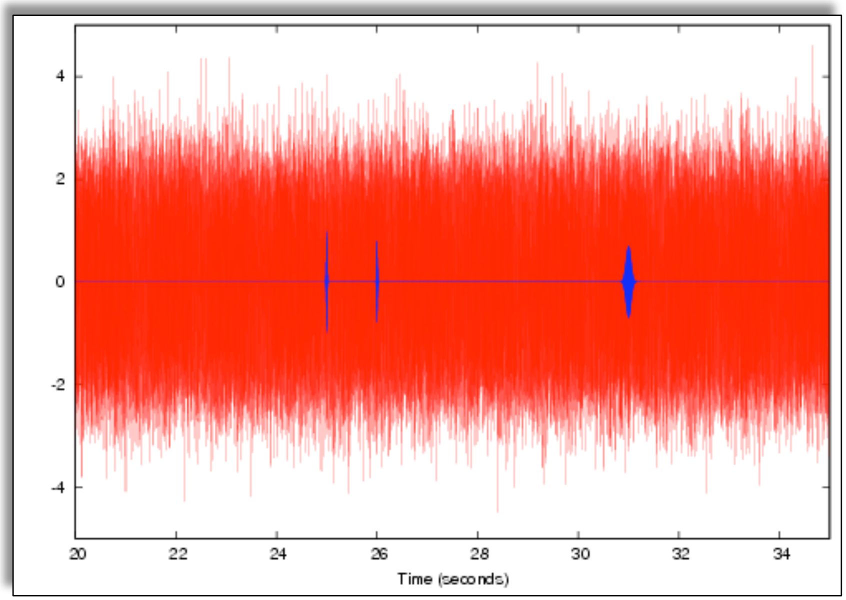

Hinder, Vaishnav, Herrmann, Shoemaker, Laguna submitted to PRL (Nov 2007)

- The eccentricity "hair" is shed almost immediately upon entering plunge.
- The final spin depends weakly with e<0.4.
- Final spin agrees with Sperhake et al.
- For larger eccentricities, there is a peak in spin, before it reduces.



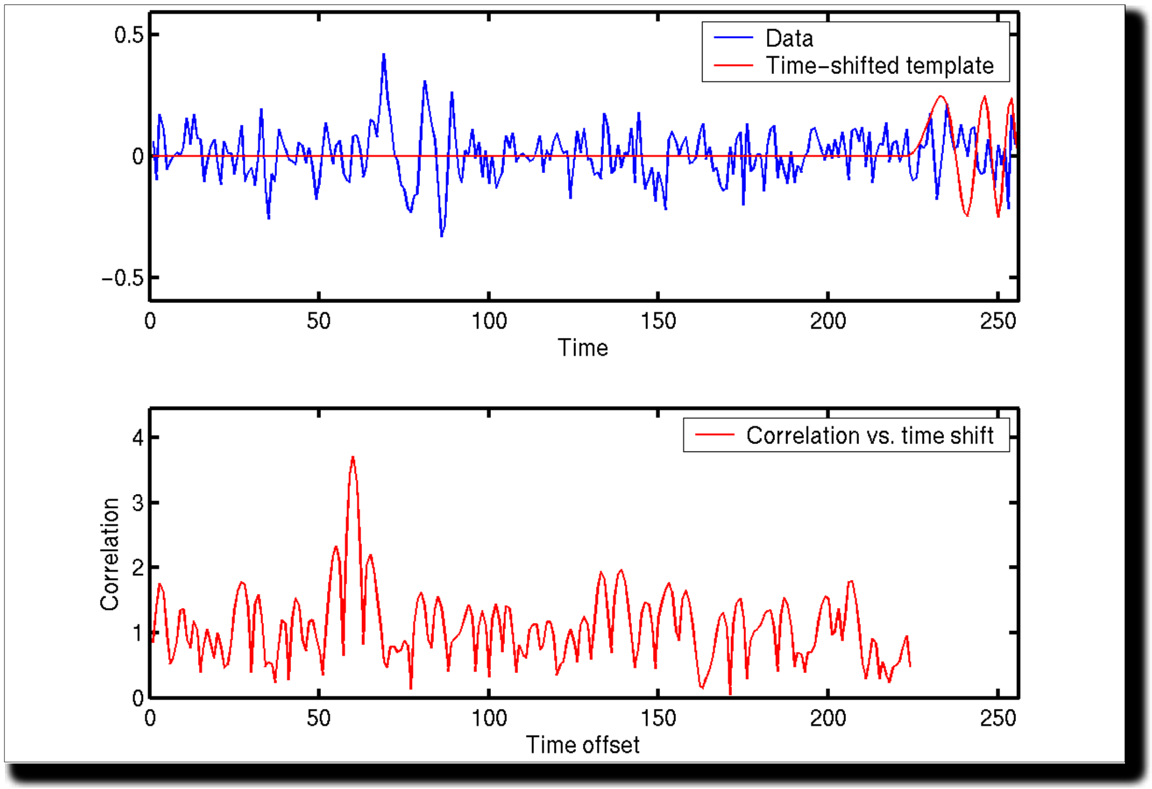
# Detection

# Noise and Signals




“The challenge in detecting GWs can be summarized as finding a needle in a haystack, except not only you first need to know where the stack of hay is, but the needle, for all you know, looks like a piece of hay” Winicour

# Matched Filter

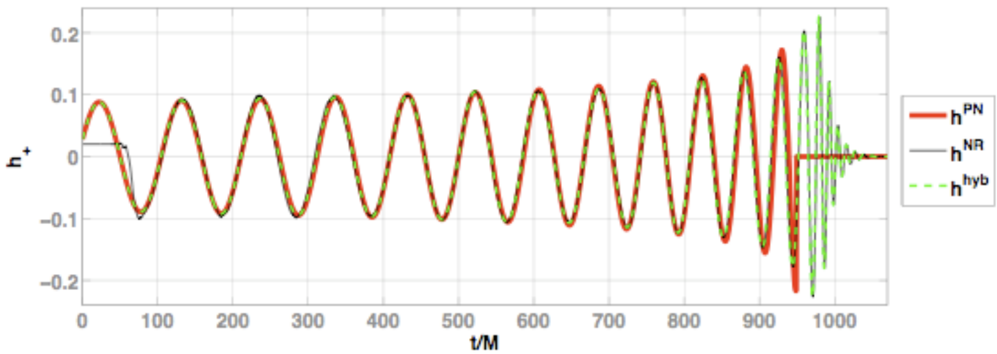



We are dominated by phase errors

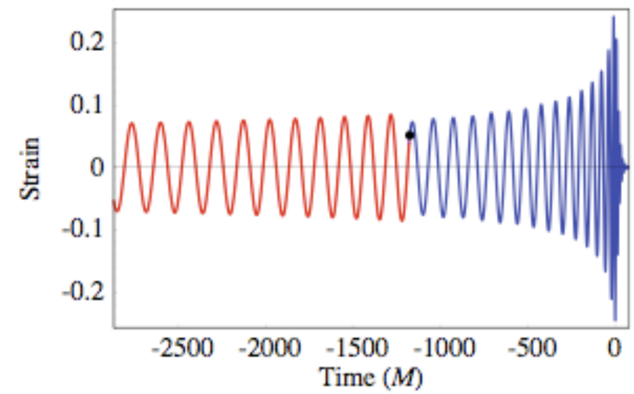
$$\langle a|b\rangle = \int_{f_{low}}^{f_{max}} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}\,df$$

# Informing Gravitational Wave Detection

Ajith et al




Pan et al



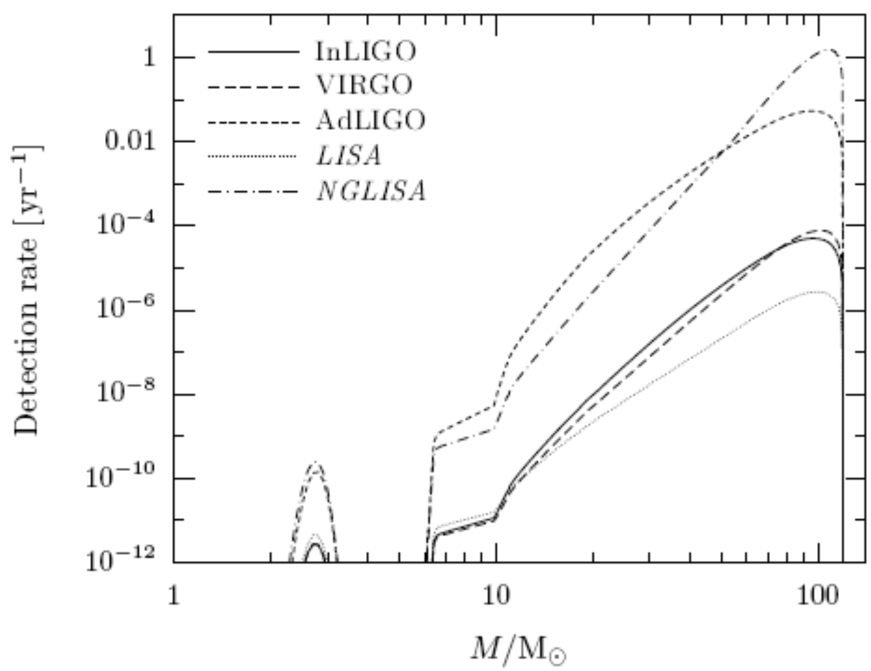

- Create Hybrid Waveforms and test analytic and hybrid template banks
  [Ajith et al arXiv.org:0704.376, Pan et al arXiv:0704.1964, Ajith 0710.2335, Buonanno et al 0706.3732]
- Test NR Waveforms in Matched Filtering
  [Buonanno et al PRD 75 (2007), Baumgarte et al gr-qc/0612100, Vaishnav et al PRD 76 (2007), Shoemaker et al 0802.4427]
- Build Template Banks & Conduct Parameter Estimation [NINJA]
- Eccentric Binaries with post-Newtonian [Martel and Poisson (1999), Tessmer, A. Gopakumar (2008) ]

# Are eccentric BBHs interesting for Detection of Gravitational Waves?

- BBHs circularize before they reach the LIGO band.
  [Gultekin, Miller, Hamilton ApJ 640 (2006) and Mandel, Brown, Gair, Miller arXiv 0705:0285]

- Some scenarios for eccentric binaries within ground based detector's reach have been suggested.
  [Kozai 1962, Miller and Hamilton 2002, Wen 2003, Campanelli et al 2008, ...]

- More scenarios suggested for LISA
  [Benacquista 2002, Guletkin et al 2004,... ]



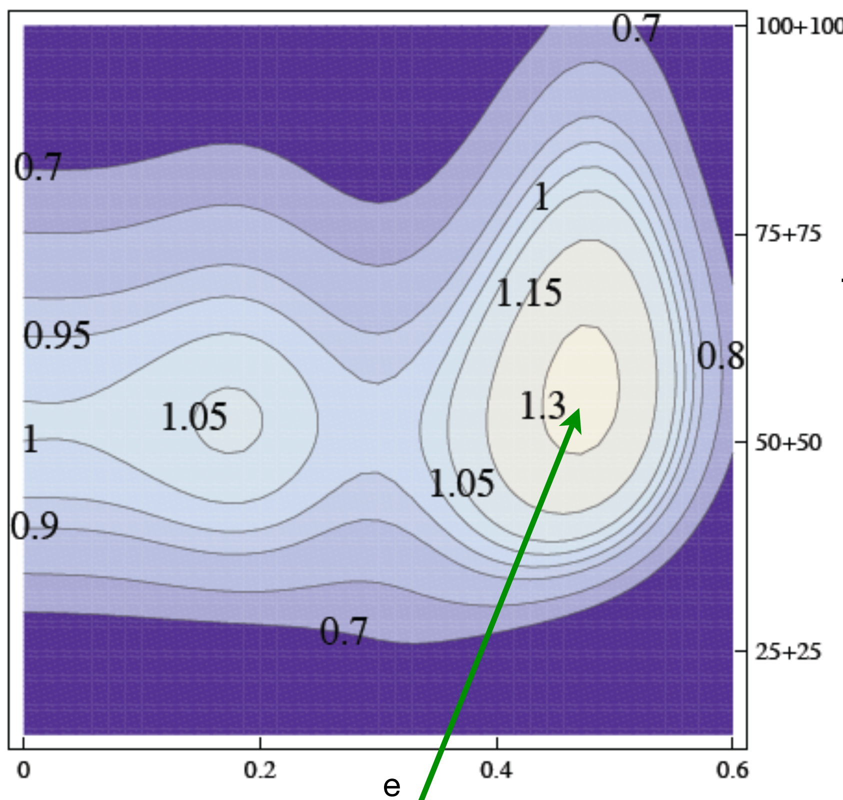

*Detection Rate Estimates of Gravity-waves Emitted During Parabolic Encounters of Stellar Black Holes in Globular Clusters,*
Bence Kocsis, Merse E. Gaspar, Szabolcs Marka Astrophys.J. 648, 411 (2006)

# How Blind are Interferometers to Eccentricity?

Signal to noise ratio from waveforms of eccentric binaries
with initial LIGO noise curve

$SNR = \langle h(f) \mid h(f) \rangle$ h(f): one of a range of numerical, eccentric waveforms

SNR
where SNR(e=0)=1




total mass
of the
binary

B. Vaishnav

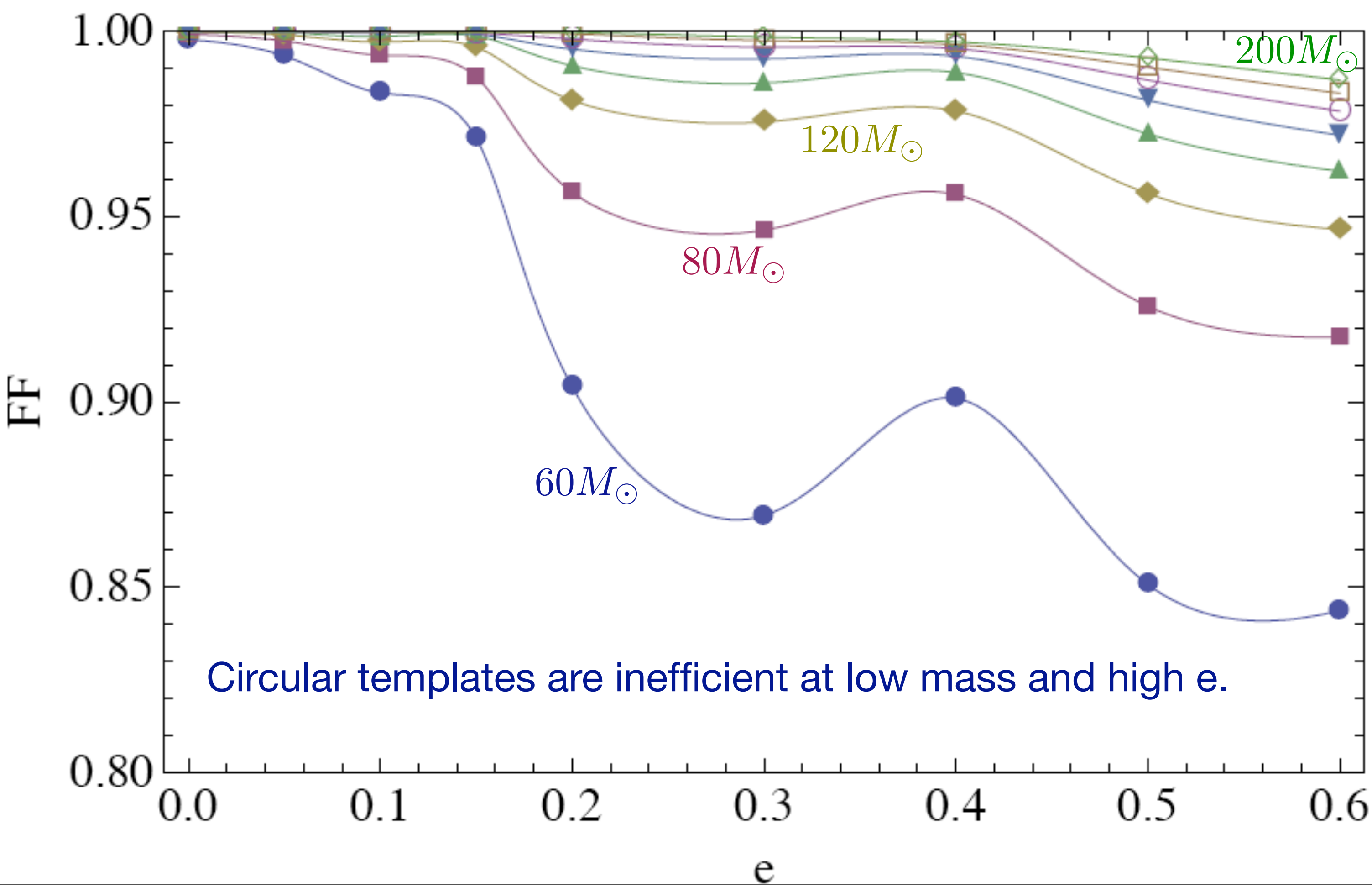
LIGO is 30% more sensitive at eccentric
binaries (e=0.45) than circular, implying a
greater reach in detection volume

# Can we see the unexpected with circular templates?

Interferometric detectors use matched filtering to optimize the detection of inspiraling signals.
If LIGO assumes no eccentric binaries will be merging, will they miss a signal?

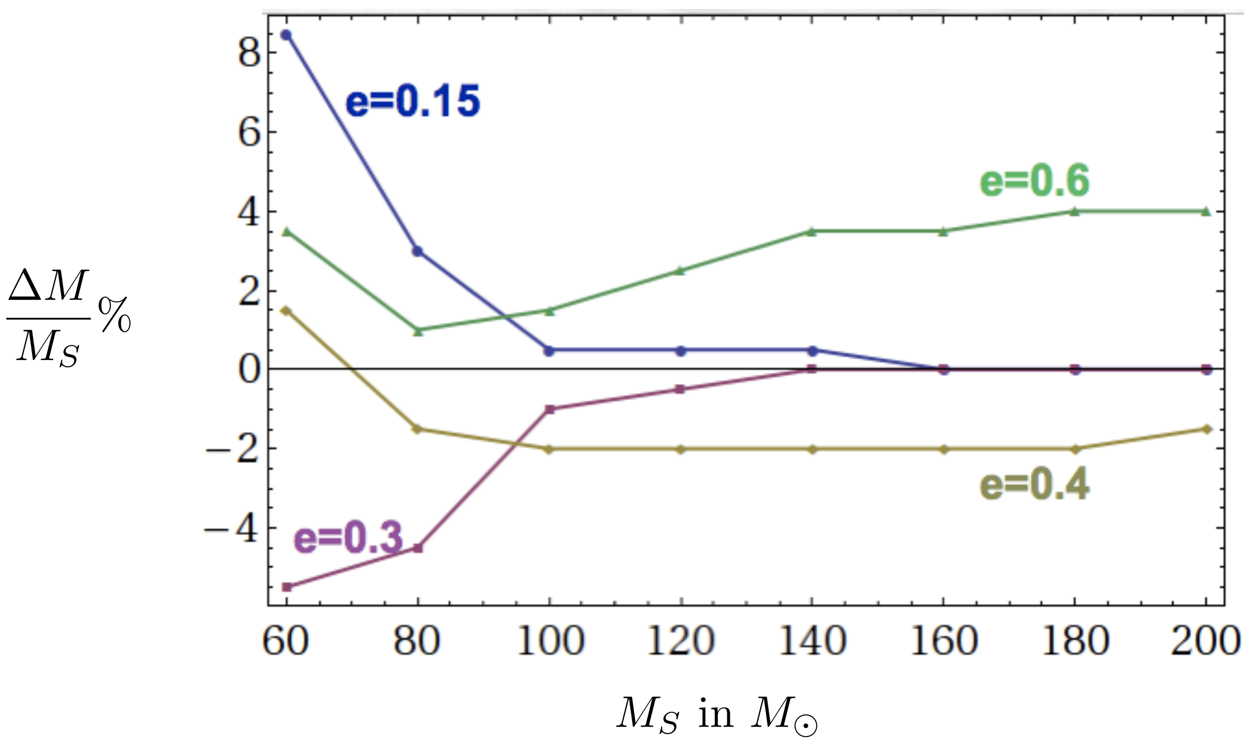
$$FF \equiv max_{m_T} min_{\phi_S} max_{t_0,\phi_T} \frac{\langle S|T\rangle}{\sqrt{\langle S|S\rangle\langle T|T\rangle}}$$

S: signal is one of a range of numerical, eccentric waveforms
T: template is a numerical, circular waveform



# Estimate of fractional mass parameter error

$$\Delta M \equiv M_S - M_T$$

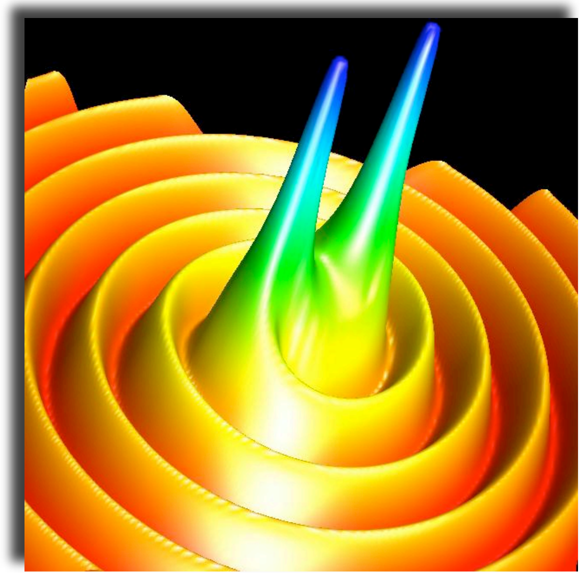



At low e, error decreases with increasing mass.
At high e, error increases with increasing mass.

# Computational Challenges

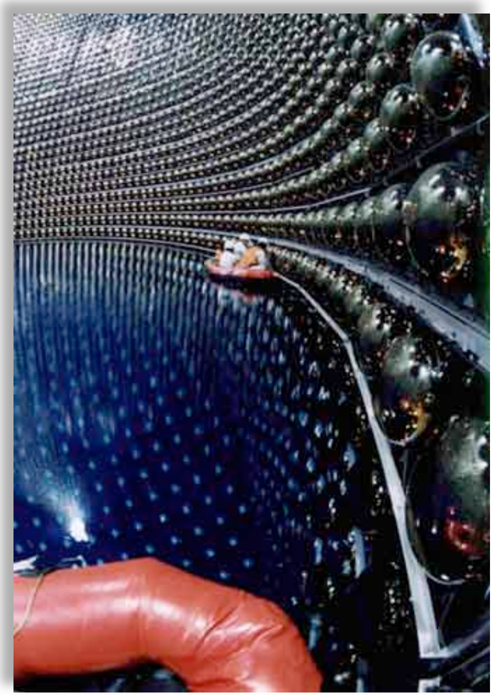
# Scalability and Bandwidth

- The most serious problem is with the AMR:
  - Cactus-Carpet
  - Samrai
  - Hahndol-Paramesh
  - BAM
- Most of current BH codes need to improve their scaling
- Memory bandwidth is already a bottleneck



# Multi-messenger Astronomy
## Center for Relativistic Astrophysics


Photons + High Energy Particles + Gravitational waves



# Conclusions for Binary Black Holes in Eccentric Orbits

- How does the binary approach its final state?
  - eccentricity sheds quickly during merger
  - peak spin of 0.72 for non-spinning, eccentric, equal-mass binaries
  - $e<0.4$ same final black hole formed
- Comparing with post-Newtonian
  - good agreement between NR and PN
  - phase difference less than 0.1 for first 8 cycles
  - good news for hybrid templates
- Are ground-based detectors blind to eccentricity for intermediate mass binaries?
  - yes for $e<0.1$ when including only dominant mode
  - chance to use LIGO to distinguish between astro models
- Computational Challenges and Future Simulations
  - Scalability
  - Multi-messenger astronomy