Dynamical Evolution of Perturbed Spherically Symmetric Boson Stars

Scott H. Hawley

Outline of Project

I will attempt to model the dynamical evolution of perturbed states of "boson stars" – equilibrium configurations of complex scalar fields.[1] These fields have a Lagrangian density given by

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\phi^*_{;\mu}\phi_{;\nu} - \frac{1}{2}m^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4,$$

where m is the boson mass and λ is some parameter. This Lagrangian gives rise to the scalar field equation

$$\Box\phi = m^2\phi + \lambda|\phi|^2\phi. \tag{1}$$

The evolution of the scalar field is completely determined by this equation and the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}, \tag{2}$$

where

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial}{\partial g_{\mu\nu}}\mathcal{L} \tag{3}$$

and $g = \det(g_{\mu\nu})$.

Assumptions:

- Spherical symmetry, so the metric is given by

  $$ds^2 = -[N(r,t)^2 - S(r,t)^2]dt^2 + 2S(r,t)dtdr + g^2dr^2 + r^2d\Omega^2,$$

  where $N(r,t)$ is called the *lapse* function and S is called the *shift*.
  (N.B.: I don't really know what these terms mean, but I intend to study the paper by York.[2])

- $\lambda = 0$, i.e self-iteraction occurs only through the mass.

- I will make use of the "polar-slicing condition", requiring $K_{00} + K_{\varphi\varphi} = 0$, where K is the extrinsic curvature of a constant t slice. This results in $S(r,t) = 0$.

Equations to solve:

- For $S(r,t) = 0$, Eq.(1) becomes

  $$-\frac{1}{N^2}\left[\ddot{\phi} - \frac{\dot{N}}{N}\dot{\phi} - \frac{NN'}{g^2}\phi'\right] + \frac{1}{g^2}\left[\phi'' - \frac{g'}{g}\phi' - \frac{g\dot{g}}{N^2}\dot{\phi}\right] + \frac{2\phi'}{rg^2} - m^2\phi = 0, \tag{4}$$

  where $' \equiv \partial/\partial r$ and $\dot{} \equiv \partial/\partial t$.

- Letting $\phi(r,t) = \Phi_1(r,t) + i\Phi_2(r,t)$, the rr component of the Einstein equation is

  $$N' = \frac{1}{2}N\left[\frac{g^2-1}{r} + 4\pi Gr\left((\Phi_1')^2 - g^2m^2\Phi_1^2 + (\Phi_2')^2 - g^2m^2\Phi_2^2 + \frac{g^2}{N^2}(\dot{\Phi}_1^2 + \dot{\Phi}_2^2)\right)\right]. \tag{5}$$

- The Hamiltonian constraint equation, $G_{tt} - 8\pi G T_{tt} = 0$, gives

$$g' = \frac{1}{2}\left[4\pi G\left(g^3 r\frac{\dot\Phi_1^2 + \dot\Phi_2^2}{N^2} + rg(\Phi_1')^2 + rg(\Phi_2')^2 + rg^3 m^2(\Phi_1^2 + \Phi_2^2)\right) - \frac{g(g^2-1)}{r}\right]. \tag{6}$$

  Solving (6) is considered computationally time-consuming, so $g(r,t)$ is calculated using the tr component of the Einstein equation:

$$\dot g = 4\pi G r g(\dot\Phi_1 \Phi_1' + \dot\Phi_2 \Phi_2'). \tag{7}$$

Equations (4), (5) and (7) will be solved numerically. Equation (5) is a first-order ODE for N, and can be solved using an ODE solver such as `lsoda`. Equations (4) and (7) are nonlinear PDEs, and might be solved either by a "homemade" scheme of my own design or using a "modified MacCormack predictor-corrector scheme" described in a book by Bernstein *et al.*[3]

Initial conditions:

An equilibrium configuration for a boson star. (In class, we discussed a method of solving for such a configuration.[4])

Boundary conditions:

- We require that spacetime is nonsingular at $r = 0$.
- Asymptotically flat spacetime as $r \to \infty$.
- From Seidel and Suen[1]: "At the outer boundary, we choose the lapse function to be fixed at its initial value."
- Scalar waves should only be *outgoing* at large radii. Seidel and Suen[1], whose methods I will follow, did observe some reflection from the grid boundaries, so this condition may not be met. They did remark that this had "practically no effect on the accuracy of the calculations."

Expectations/Hypotheses:

It is expected that boson stars will be unstable beyond a certain total mass.

References:

[1] E. Seidel and W.-M. Suen, *Phys. Rev. D*, **42**, 384, 1990. (This work will be followed closely.)

[2] J.W. York, Jr., in *Sources of Gravitational Radiation,* proceedings of the Battelle Seattle Workshop, 1978, Larry Smarr, ed. Cambridge University Press, Cambridge, England, 1979.

[3] D.H. Bernstein, D.W. Hobill and L.L. Smarr, *Frontiers in Numerical Relativity*, Cambridge University Press, Cambridge, England, 1989.

[4] M. Colpi, S.L. Shapiro and I. Wasserman, *Phys. Rev. Lett.*, **57**, 2485, 1986.