Black Hole–Scalar Field Interactions in Spherical Symmetry

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Overview

- Coordinate System
- Equations
- Power-law Tails
- *Quasi-normal* Ringing
- Definition of *Criticality*
- Mass Scaling
- Non-linear Effects
- Coordinate Effects

Coordinate System

Minimally-Modified Ingoing Eddington-Finkelstein Coordinates (M.W. Choptuik and W.G. Unruh)

$$ds^2 = \left(-\alpha^2 + a^2\beta^2\right) dt^2 + 2a^2\beta dt dr + a^2 dr^2 + r^2 b^2 d\Omega^2$$

$$s = r + f\left(t\right)$$

$$ds^2 = \left(-\alpha^2 + a^2\beta^2\right) dt^2 + 2a^2\beta dt dr + a^2 dr^2 + s^2 d\Omega^2$$

Slicing:

$$\vec{\partial}_t - \vec{\partial}_r \text{ is null}$$

$$g_{tt} - 2g_{tr} + g_{rr} = 0$$

$$\alpha = a\left(1 - \beta\right)$$

Shift:

$$\dot{s} = -\alpha s K^\theta{}_\theta + \beta s'$$

$$\beta = \dot{f} + s\alpha K^\theta{}_\theta$$

$$\beta = \frac{\dot{f} + saK^\theta{}_\theta}{1 + saK^\theta{}_\theta}$$

Equations

Scalar Field:

$$\Pi \equiv \frac{1}{1-\beta}\left(\dot{\phi} - \beta\phi'\right) \qquad \Phi \equiv \phi'$$

$$\dot{\Pi} \equiv \frac{1}{s^2}\left(s^2\left(\beta\Pi + \left(1-\beta\right)\Phi\right)\right)' - 2\,\frac{\dot{s}}{s}\,\Pi$$

$$\dot{\Phi} \equiv \left(\beta\Phi + \left(1-\beta\right)\Pi\right)'$$

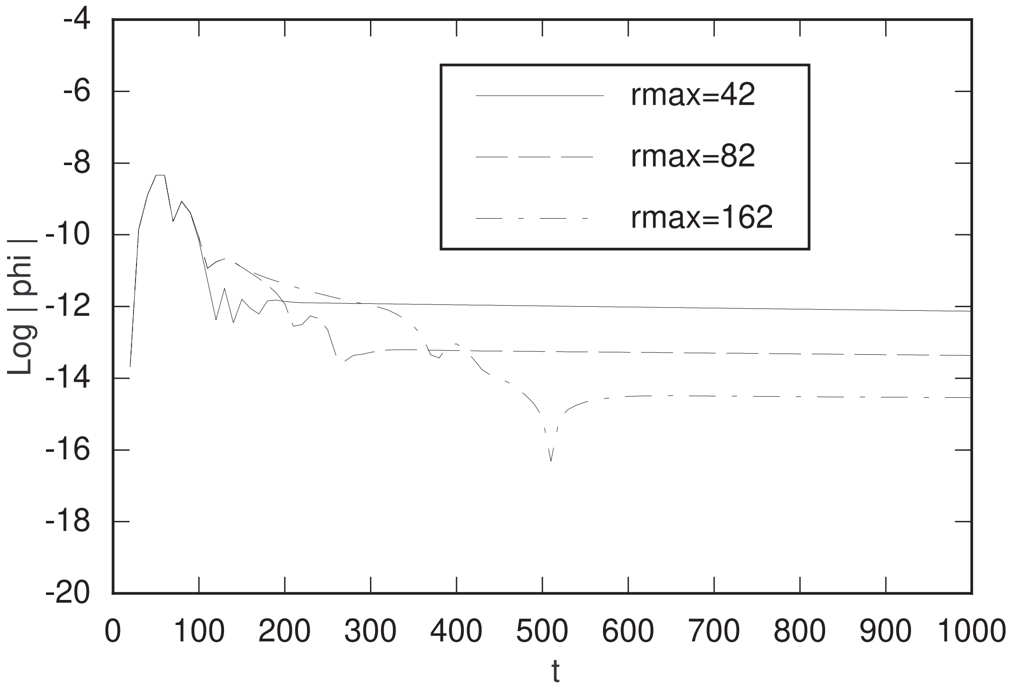
Geometry:

$$\dot{a} = -a^2\left(1-\beta\right)K^r{}_r + \left(a\beta\right)'$$

$$K^\theta{}_{\dot{\theta}} = \beta K^\theta{}_\theta{}' + a\left(1-\beta\right)K^\theta{}_\theta\left(K^r{}_r + 2K^\theta{}_\theta\right) + \frac{1-\beta}{s^2}\left(a - \frac{1}{a}\right) + \frac{\beta'}{as}$$

$$K^r{}_r = sK^\theta{}_\theta{}' + K^\theta{}_\theta - \frac{4\pi s\Phi\Pi}{a}$$

Power-law Tails

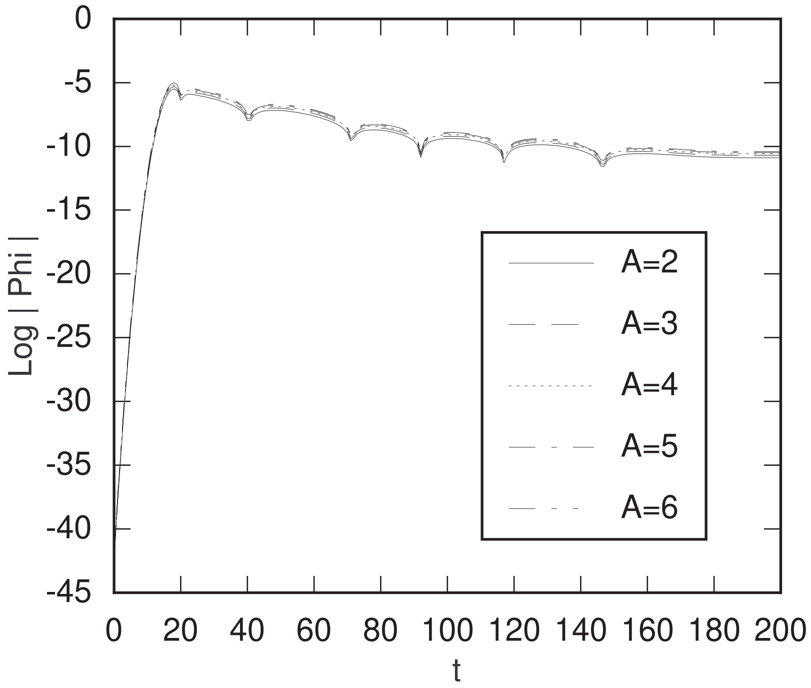


Log ϕ at $r = 30$ verses t

The power-law fit:

$$\phi(t) = at^{-3.38}$$

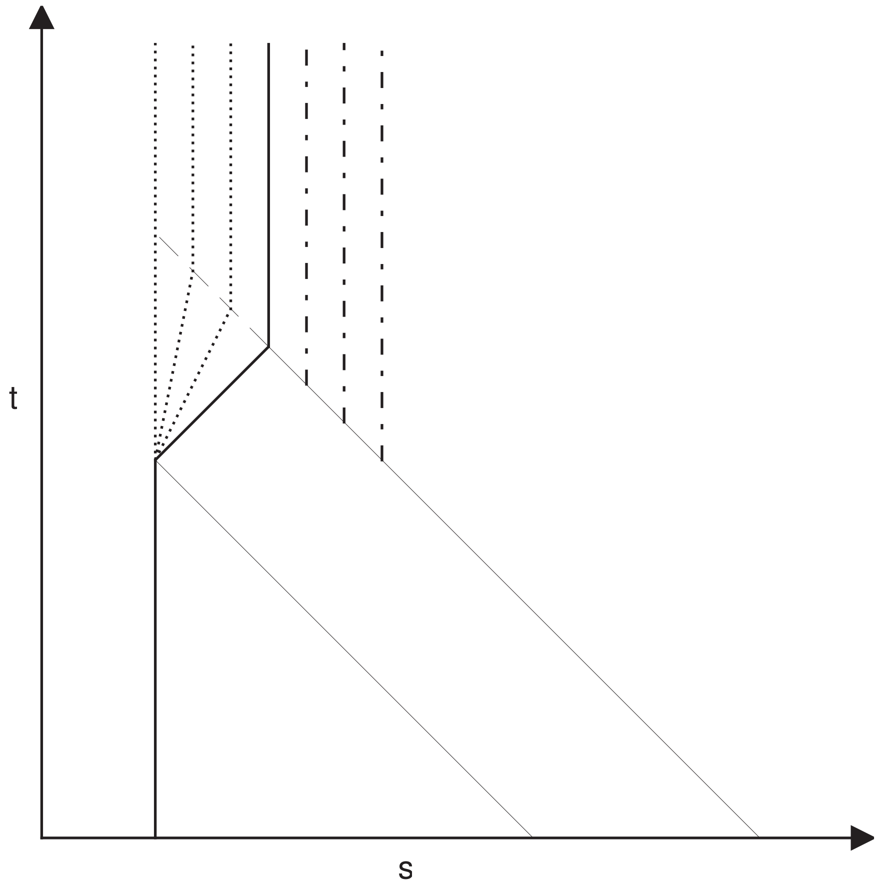
Quasi-normal **Ringing**



Log ϕ at the horizon verses t for various amplitudes

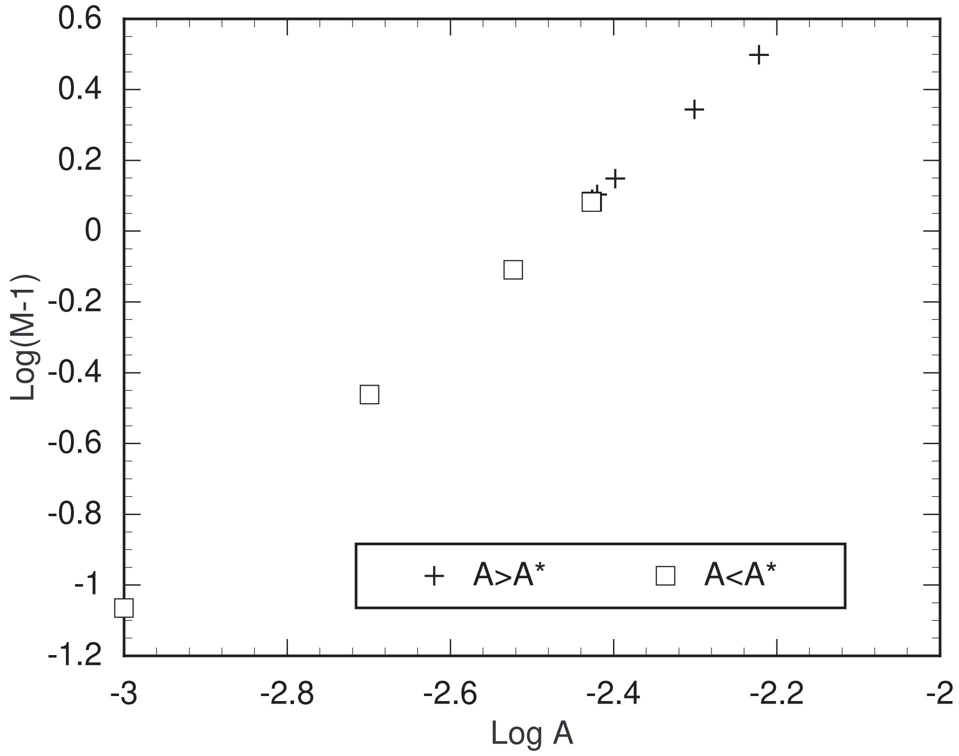
The period for one oscillation is $\approx 53M$

Definition of *Criticality*



Schematic motion of the horizon for various amplitudes of scalar field

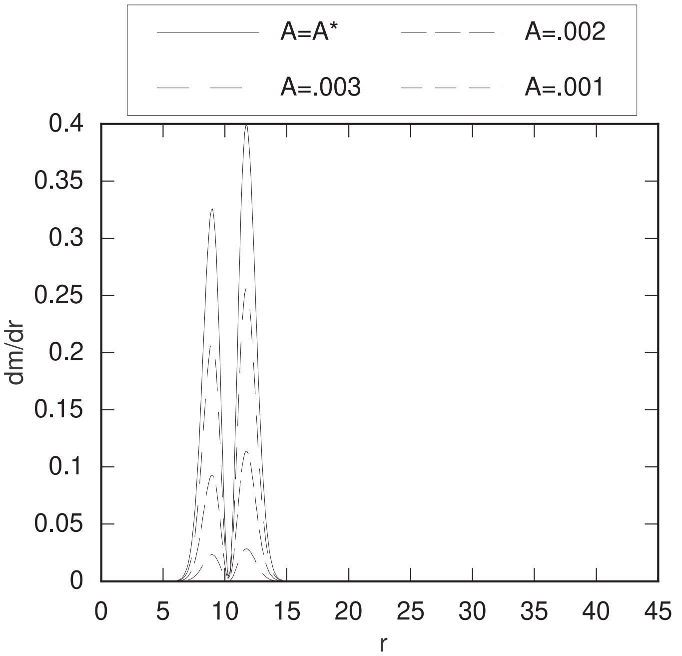
Mass Scaling

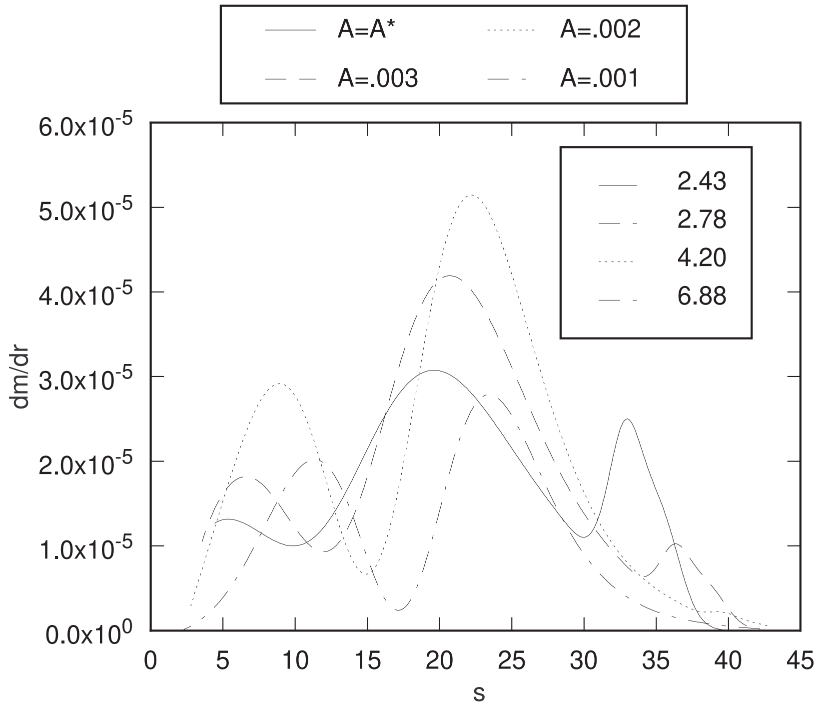


Mass verses amplitude

The graph is fit by a line with slope 2.01.

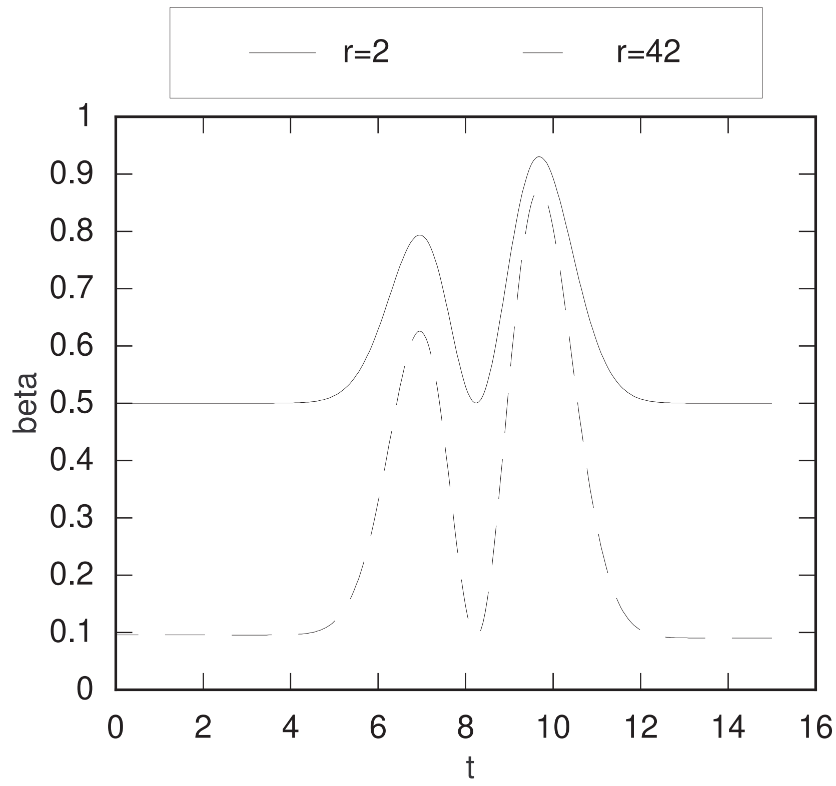
Non-linear Effects





The inset shows 1000 times the fraction of mass scattered.

Coordinate Effects



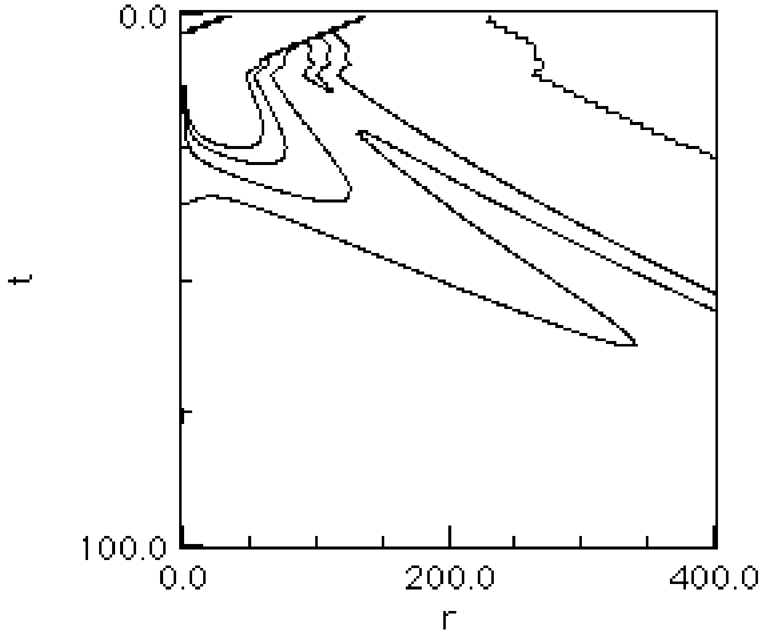
Shift:

$$\beta = \frac{\dot{f} + saK^{\theta}{}_{\theta}}{1 + saK^{\theta}{}_{\theta}}$$

At the horizon:

$$\beta = \frac{\dot{f}}{2} + \frac{1}{2}$$

Characteristic speeds: $-1,\ 1 - 2\beta$



Evolution of $\frac{dM}{dr}$ showing retrograde motion