Chapter 1

Introduction

Early studies of Einstein's theories of gravity had to be content with investigating weak field approximations and spacetimes with high degrees of symmetry due to the fact that the Einstein equations are a complicated, non-linear system of differential equations involving second-order derivatives of the metric components. Despite this difficulty, a great deal was learned about the nature of gravitational spacetimes via analytical methods. The spherically symmetric Schwarzschild spacetime was not only the first solution to the Einstein equations, but it also indicated physical phenomenon such as the existence of event horizons and the gravitational red-shift effect. The study of the linearized Einstein equations indicated the existence of gravitational waves. These predictions of unusual phenomonen were exciting, but the study of strong-field phenomenon associated with astrophysical situations likely to occur within nature were very intractible, if not impossible, through analytical methods.

With the continuing increase in supercomputer performance, the study of dynamic, strong-field spacetimes is becoming more and more approachable. Computational techniques, when rigorously applied, have proven to be a powerful theoretical laboratory where the implications of a complex physical theory can be applied [2]. A significant study was accomplished nearly twenty years ago by Smarr [5] in his numerical solution of an axially-symmetric binary black hole collapse. Through evolving two-dimensional (2D), spatial hypersurfaces, Smarr showed the coalescence of the two event horizons into a single event horizon and was able to estimate the gravitational radiation released. Additional studies have revealed.....

Since...... , computer codes capable of evolving three-dimensional (3D) hypersurfaces has been pursued by many researchers. While significant results have been obtained, a general numerical technique for evolving strong-field spacetimes has proved quite elusive.

The goal of this research was to develop a computer code suitable for studying the properties of arbitrary binary black hole spacetimes. The Arnowitt-Desser-Missner (ADM) theory and the York formalism provides a now standard theoretical scheme for recasting the Einstein equations into a nearly hyperbolic form where 3D spacelike hypersurfaces can be evolved dependent upon a timelike parameter.

The domain of the physical spacelike hypersurfaces extends out to infinity, but the computational domain for these hypersurfaces must be circumscribed within a finite volume. Additionally, the presence of the singularities associated with the black holes requires us to avoid computing field values too near the singularities less we abort the computation due to floating point overflow. An obvious course of action is to excise the region near the singularity (ref......). Thus our computational domain will be cubic or rectangular with one or more holes cut out to exclude the singularities as illustrated in figure (1.1). It is important to note these boundaries to our computational domain are entirely artificial. That is, we can not rely upon physical boundary conditions which

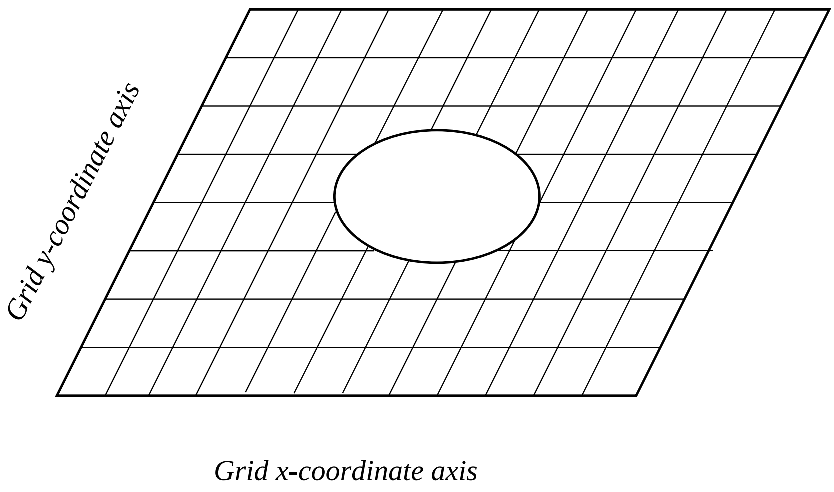

Figure 1.1: A 2D cross section of the 3D computational grid (z-axis suppressed). The outer rectangle marks the finite extent of the computational grid and the inner boundary excises the region surrounding the singularity associated with a black hole.

would determine field values at these boundary grid points. This turns out to be a significant complication to our problem. It has proven difficult to find a stable numerical scheme for grid points which don't have nearest neighbors on all sides. Indeed, the CFL theory seems to show that numerical solutions of hyperbolic equations using one-sided difference equations violated causality and would lead to non-convergent solutions. Often, in numerical solutions to time-evolution problems, one can invoke other physical conditions which provide an alternative numerical update scheme, such as an outgoing wave condition which can be used in evolutions of flat-space, linear waves. However, these techniques have proven limited in their application to black hole spacetimes.

A major part of this work is devoted to the problem of finding stable update schemes at the inner and outer boundaries. A general method is preferred; one that suffices at the outer boundary and also the boundaries of the excised holes. A variety of update schemes were investigated and the problems and relative merits are described. The common failure of these boundary updates is often explained by their violation of causality as described by the Courant-Freidrich-Levy condition. The appropriateness of this analysis is also explored.

???? have used excised regions that correspond to apparent horizons which are spacelike, minimally-trapped surfaces surrounding spacetime singularities. With this approach, one can attempt to use causality arguments which will suggest an additional mathematical condition which can be implemented in an update scheme. The merit of these approaches is also studied.

The interpretations of the success and failure of many of these techniques which appear in the literature are doubtful. It is tempting, and common in the literature, to ascribe an ad hoc hypotheses to observed instabilities in the solution based on a physical or causal mechanism. However, many of these hypotheses are not verified via further experimentation and critical analysis. The acceptance of these explanations can lead the researcher to try unfounded approaches for a successful scheme or even worse, cause one to wrongly dismiss potentially successful approaches. Most likely, instability in evolving causal spacetimes is simply due to the algebraic structure in the Banach space (??????) of the numerical method use to update one time slice to the next. In the end, obtaining stable numerical evolutions is both an art and a science, and a judicious use of both creativity and rigor are needed.

To find suitable boundary update schemes, several prototype versions of a black hole evolution code are used. These prototype codes are first applied to single black hole spacetimes. These codes provided insight into the nature and difficulty of the problem. Grid boundary updates are made using a variety of techniques based on extrapolating values from the interior. While it was clear the extraplotion techniques were inherently unstable, these evolutions of Schwarzschild spactimes were useful in gaining experience with finite differencing the Einstein equations and showing that standard finite difference scemes for interior grid points were straightforward and completely stable. Evolutions of single black hole spacetimes express in time-dependent Novikov coordinates again showed that standard finite difference techniques were adequate for interior regions, while the updates of grid points at the outer boundary and the excised singularity were unstable using extrapolation techniques.

Additionally, to avoid dealing with singularities in the spacelike hypersurfaces, a version of the black hole evolution code was run using initial data for weak gravitational waves. This removed the complication of excising the singularity and dealing with the instabilities common at this inner boundary. Again it was found that weak waves could be stably evolved through the space time until they propogated to the outer boundary where the one-sided update schemes were unstable.

An important aspect of the stability problem is to realize that the biggest difficulty comes from a computational boundary that is not a physical boundary and not from the complexity of the Einstein equations. To gain insight into this essential problem, a model code for evolving flat-space, linear waves was used to conveniently implement and demonstrate the validity of proposed boundary update schemes. Here new finite difference stencils and corresponding algebraic update equations could be easily implimented without the complication of revising the finite differencing of the Einstein equations and worrying about the effect of curved spacetime dynamics. Although success of a finite difference update scheme in the model problem will not guarantee success in evolving black hole spacetimes, failure to evolve linear waves in flat space almost certainly means the method will not be able to stably evolve dynamical features in a black hole spacetime.

The more promising evolution schemes were then implemented in a sophisticated code being developed in a collaborative effort by the Binary Black Hole Grand Challenge[1]. This code incorporates apparent horizon tracking techniques for recomputing excised regions for black holes as they move through the spacelike hypersurface. It also incorporates advanced computer programming techniques to take advan tage of massively parallelized computing architectures....... Evolutions of single black hole spacetimes, binary black hole spacetimes, and non-singular spacetimes with gravitational waves were completed. The results show....

[1]The Grand Challenge Alliance is an NSF funded collaboration among physicists and computer scientists at eight institutions whose goal is to develop numerical codes to solve the problem of the 3D spiraling coalescence of two black holes. (http://jean-luc.ncsa.uiuc.edu/GC/GC.html)