Project #2: Moving Data, Analytic Solution

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1 Non-Zero Initial Velocity

Now we set up the initial conditions such that the Gaussian pulse has an initial velocity of 1.0. One does this by shifting the pulse to the left by a distance of Δt and using the shifted pulse to define step n =-1. Figure 1 shows the initial pulse generated by the program Project02.f, with $\Delta x = 0.005$, $\Delta t = 0.0045$, and the Gaussian width $2\Delta = 0.08$. Figure 2 shows that the evolution results in the pulse moving to the right with a velocity of 1.0. A fixed boundary reflects the pulse back with a negative sign in Figure 3. The pulse reappears on the left side for a periodic boundary (Figure 4). And in Figure 5 (radiation boundary), the pulse disappears off the screen.

2 Analytic Solution

This time, the program Converge02.f uses the analytic solution Qa of the wave equation to calculate the convergence factor. The following figures use $\Delta x = 0.0025$ and $\Delta t = 0.00125$ for the coarsest grid $4h$. For each timestep n, the program calculates Q_a and uses it to calculate the norms for grids $Q4$ ($\Delta x = 4h$), $Q2$ ($\Delta x = 2h$), and $Q1$ ($\Delta x = h$).:

$$
\begin{aligned}
\|Q4^n - Qa^n\| &= \sqrt{\frac{\sum_j (Q4_j^n - Qa_j^n)^2}{2 * Nx + 1}} \\
\|Q2^n - Qa^n\| &= \sqrt{\frac{\sum_j (Q2_j^n - Qa_j^n)^2}{4 * Nx + 1}} \\
\|Q1^n - Qa^n\| &= \sqrt{\frac{\sum_j (Q1_j^n - Qa_j^n)^2}{8 * Nx + 1}}
\end{aligned}
$$

where $Nx = 1/4h$. The program then prints out graphs of the two versions of the convergence factor.

$$
\begin{aligned}
CF42 &= \frac{\|Q4^n - Qa^n\|}{\|Q2^n - Qa^n\|} \\
CF21 &= \frac{\|Q2^n - Qa^n\|}{\|Q1^n - Qa^n\|}
\end{aligned}
$$

Figures (6) and (7) correspond to the Periodic boundary, Figures (8) and (9) to the Fixed boundary, and Figures (10) and (11) to the radiation boundary. The use of the analytic solution eliminates the spikes seen in the numeric calculations of the convergence factor.