Tracking Apparent Horizons via Level Set Method

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Outline

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Motivation

Physical

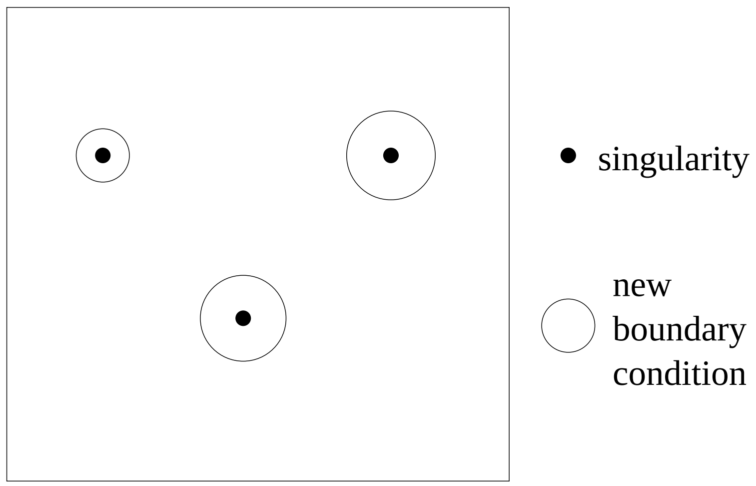
⋆ Test General Relativity in Strong Field

⋆ Detect Gravitational Radiation

Numerical

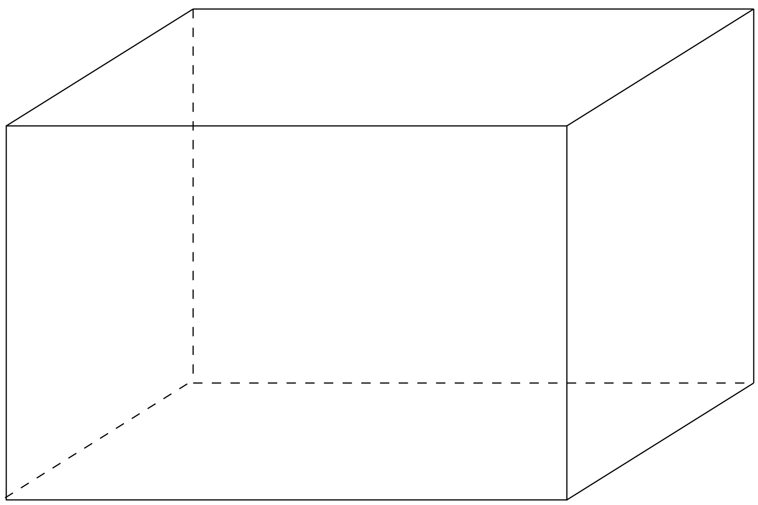
⋆ Identify Black Holes in Domain

⋆ Create Inner Boundary Conditions



Apparent Horizons are "Natural" Boundaries

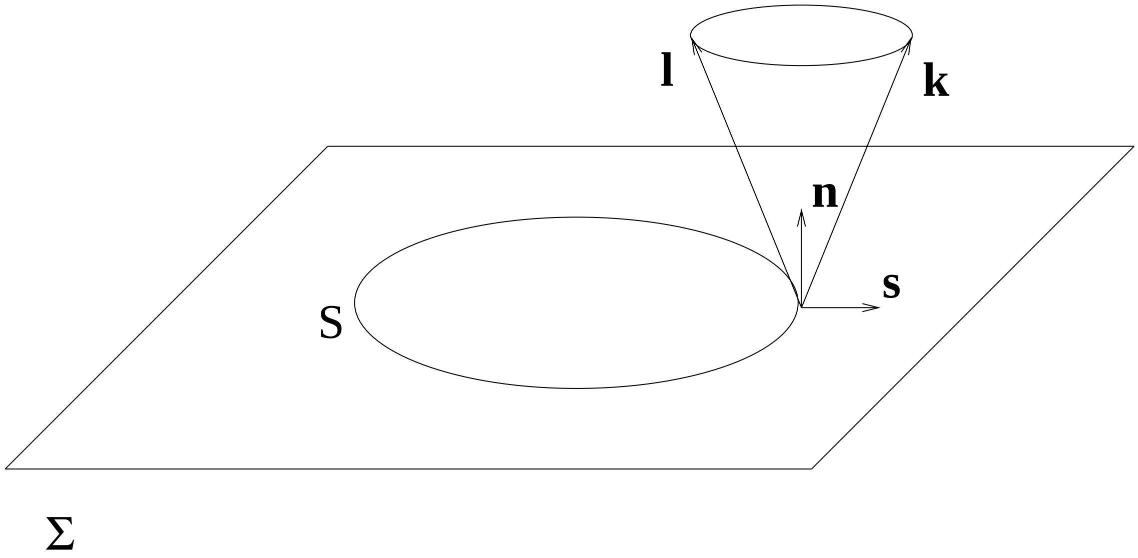
- $\star$ local
- $\star$ no information loss



Given Σ

- $\star$ K_{ij}
- $\star$ g_{ij}

Background



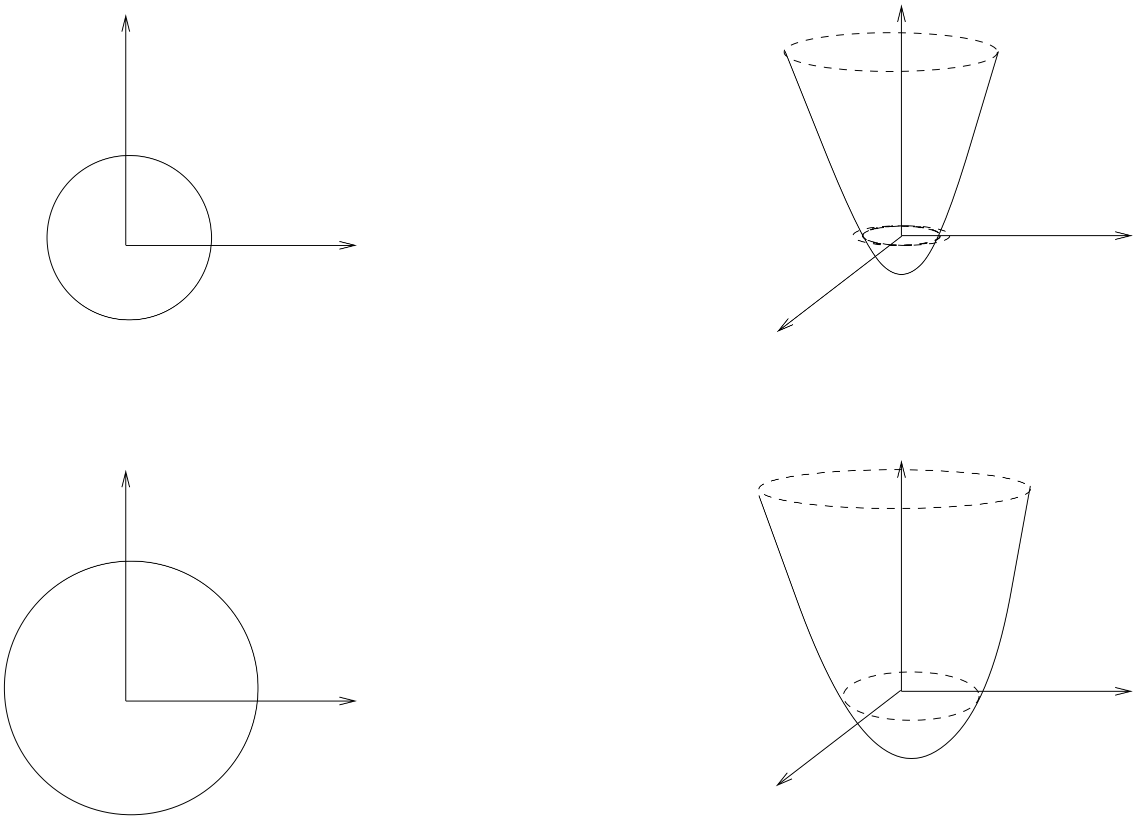
Apparent Horizons Equation

$$\begin{aligned} \kappa &= D_i s^i + K_{ij} s^i s^j - K \\ &= 0 \end{aligned}$$

2-Sphere

$$\tau = r - h(\theta, \phi)$$

Level Set Method

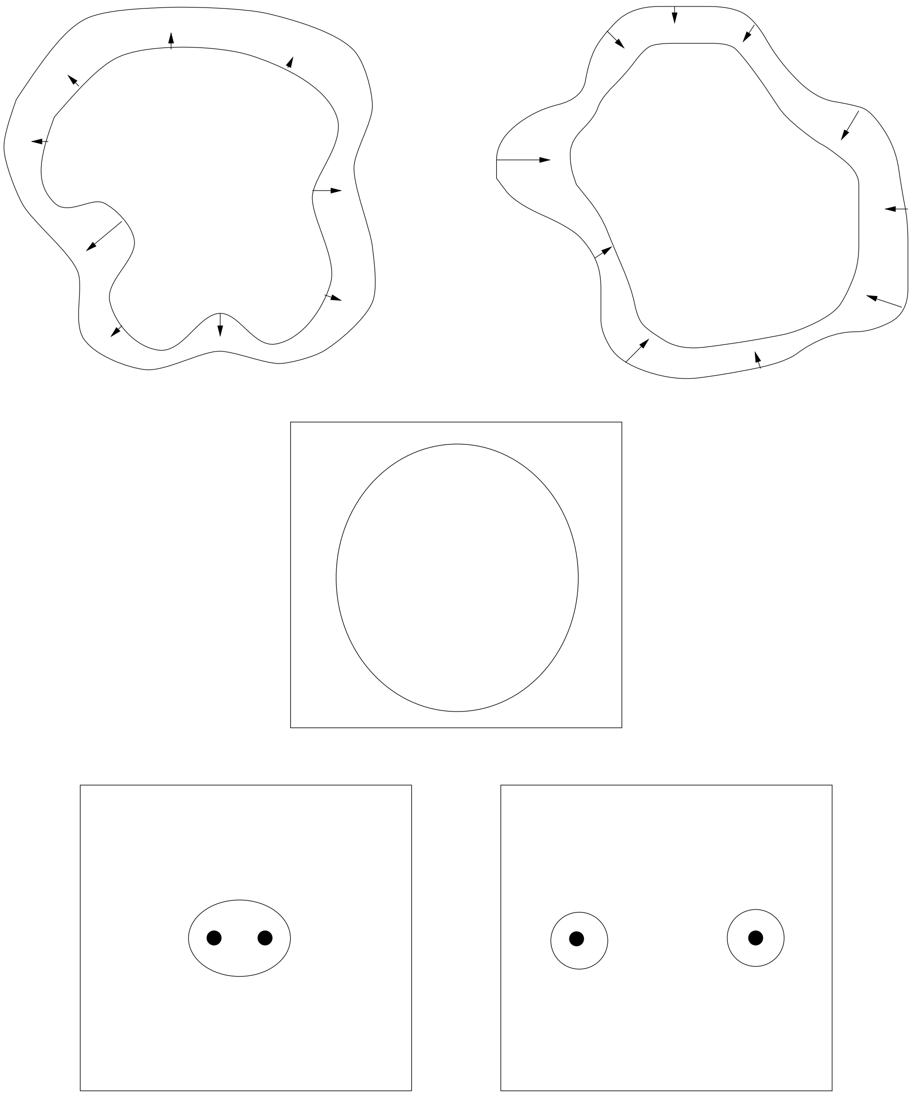


$$
\begin{aligned}
\psi(x(t),t) &= 0 \\
\psi_t + \nabla\psi \cdot x_t &= 0
\end{aligned}
$$

$$
\begin{aligned}
F &= n \cdot x_t \\
n &= \frac{\nabla\psi}{|\nabla\psi|}
\end{aligned}
$$

$$\psi_t + F|\nabla\psi| = 0$$

Level Set & Apparent Horizons



$$\Psi_t = -F(AH)\nabla\Psi$$
$$F(AH) = AH - c$$

Results

3-D Code with Crank-Nicholson Differencing Scheme

$$\hat{\psi}_i^{n+1} = \hat{\psi}_i^n - \Delta t(\hat{AH} - c)|\nabla\hat{\psi}|_i$$

Conformal Metric

$$\begin{aligned} ds^2 &= \psi^4 dr^2 + \psi^4 r^2 d\theta^2 + \psi^4 r^2 \sin^2\theta d\phi^2 \\ \psi &= 1 + \frac{M_1}{2r_1} + \frac{M_2}{2r_2} \end{aligned}$$

Conclusions & Beyond

Have

- ⋆ Full 3-D Code
- ⋆ Free Initial Surface
- ⋆ Break Capabilities at Any Orientation
- ⋆ Progress Towards Holes "Anywhere"

Want

- ⋆ Runs with Evolution Code Data
- ⋆ Speed
- ⋆ "Complete" Generality for 2 Black Holes

Beyond? Applications!