initial function:

$$\gamma(0) = (0.1 + 0.065sin(14\pi s))(cos(2\pi s), sin(2\pi s), \tag{0.1}$$

$$0 \leq s \leq 1 \tag{0.2}$$

speed function:

$$F(K) = -K \tag{0.3}$$

mean curvature:

$$K = (\psi_{yy}\psi_x^2 - 2\psi_x\psi_y\psi_{xy} + \psi_{xx}\psi_y^2)/(\psi_x^2 + \psi_y^2)^{3/2} \tag{0.4}$$

equation of motion:

$$\psi_t + F_0 H(\nabla\psi) + F_1(K)H(\nabla\psi) = 0 \tag{0.5}$$

$$F_0 = 1 \tag{0.6}$$

$$F_1 = 0 \tag{0.7}$$

The numerical method employed by Osher and Sethian in their 1989 paper is a 2nd order, essentially non-oscillatory (ENO) upwind scheme. These ENO schemes were devised for conservation laws which, when placed in a Hamilton-Jacobi framework, are supposedly simplified. In this example, the PDE contains only the convection like term, F(K) = 1; therefore, only this method is necessary. For the more general funcion, the convection term and parabolic terms are evaluated separately. The scheme they devised is based upon the Godunov scheme but is approximate instaead of exact.

$$\frac{\partial}{\partial t}\psi_j = -g_{god}(\frac{\mathrm{d}}{\mathrm{dx}}\mathrm{R^n}(\mathrm{x_j^-};\psi), \frac{\mathrm{d}}{\mathrm{dx}}\mathrm{R^n}(\mathrm{x_j^+};\psi)) \tag{0.8}$$

is the semi-discrete formulation for the approximation. To first order, this scheme yields, $u_t = -u_x$. For higher order conservation form:

$$\frac{\partial u_j}{\partial t} = -D_-[\mathrm{u_j} + \frac{\Delta\mathrm{x}}{2}\mathrm{m}[\mathrm{D_-u_j}, \mathrm{D_+u_j}]] \tag{0.9}$$

and (0.10)

$$\frac{\partial \Psi_j}{\partial t} = -[\mathrm{D_-}\Psi_\mathrm{j} + \frac{\Delta\mathrm{x}}{2}\mathrm{m}[\mathrm{D_-D_-}\Psi_\mathrm{j}, \mathrm{D_-D_+}\Psi_\mathrm{j}]], \tag{0.11}$$

where (0.12)

$$m[x,y] = x \text{ if } |x| \leq |y| \tag{0.13}$$

$$m[x,y] = y \text{ if } |x| > |y| \tag{0.14}$$

$$m[x,y] = 0 \text{ if } xy < 0 \tag{0.15}$$

for the Hamilton-Jacobi formulation. The operators are $D_{u_j} = u_j - u_{j-1}$ and $D_+ u_j = u_{j+1} - u_j$. In higher dimensions the scheme becomes more complicated. To obtain a fully discrete algorithm with the correct accuracy in time, a non-linear evolution operator is implemented such as:

$$\Psi_{jk}^{n+1} = \Psi_{jk}^{n} - \Delta t L[\Psi^n, j, k]. \tag{0.16}$$

To put the problem in terms of conservation laws we have:

$$\begin{aligned} \psi_{new} &= \psi - \Delta/2g(\psi, j, k) - \Delta/2g(\psi - \Delta t g(\psi, j, k); j, k) && (0.17)\\ \psi_{new} &= \psi - \Delta/2g - \Delta/2(1/4((u_1 + u_2) - g)^2 && (0.18)\\ &+ 1/4((u_3 + u_4) - g)^2)^(1/2). && (0.19) \end{aligned}$$