Appendix C

# Diagonal Metric Formulae

The expressions defining the Christoffel symbols and the Ricci tensor components can be greatly simplified for the special case where the metric is diagonal. This appendix presents results given by Landau and Lifshitz.[1] In this appendix the usual convention that repeated indices are summed is dropped, and all sums are made explicit with the $\sum$ symbol.

The metric is first expressed in the form

$$g_{\alpha\beta} = \delta_{\alpha\beta}\epsilon_\alpha e^{2f_\alpha}, \tag{C.1}$$

where $\epsilon_\alpha$ is the sign of $g_{\alpha\beta}$, e.g. $e_0 = -1, e_1 = e_2 = e_3 = 1$. The Christoffel symbols are

$$\Gamma^\alpha{}_{\beta\gamma} = \delta_{\beta\alpha} f_{\alpha,\gamma} + \delta_{\alpha\gamma} f_{\alpha,\beta} - \delta_{\beta\gamma}\epsilon_\alpha\epsilon_\beta e^{2(f_\beta - f_\alpha)} f_{\beta,\alpha}. \tag{C.2}$$

This expression can be simplified by examining three separate cases

$$\Gamma^\alpha{}_{\alpha\beta} = f_{\alpha,\beta} \tag{C.3}$$

$$\Gamma^\alpha{}_{\beta\beta} = -\epsilon_\alpha\epsilon_\beta e^{2(f_\beta - f_\alpha)} f_{\beta,\alpha} \qquad (\alpha \neq \beta) \tag{C.4}$$

$$\Gamma^\alpha{}_{\beta\gamma} = 0 \qquad (\alpha \neq \beta \neq \gamma). \tag{C.5}$$

The Ricci tensor components are similarly divided into two cases

$$R_{\alpha\beta} = \sum_{\mu\neq\alpha,\beta} \left( f_{\mu,\beta} f_{\beta,\alpha} + f_{\alpha,\beta} f_{\mu,\alpha} - f_{\mu,\alpha} f_{\mu,\beta} - f_{\mu,\alpha\beta} \right) \quad (\alpha \neq \beta) \tag{C.6}$$

$$R_{\alpha\alpha} = \sum_{\mu\neq\alpha} \Bigg[ f_{\alpha,\alpha} f_{\mu,\alpha} - f_{\mu,\alpha}^2 - f_{\mu,\alpha\alpha} + \epsilon_\alpha\epsilon_\mu e^{2(f_\alpha - f_\mu)} \times \left( f_{\mu,\mu} f_{\alpha,\mu} - f_{\alpha,\mu}^2 - f_{\alpha,\mu\mu} - f_{\alpha,\mu} \sum_{\nu\neq\alpha,\mu} f_{\nu,\mu} \right) \Bigg]. \tag{C.7}$$

[1] Landau L D and Lifshitz E M 1975 *The Classical Theory of Fields* (Oxford: Pergamon Press) p. 267.

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